20 Oct. 2014 – Due MONDAY, 27 Oct. 2014 in Dropbox
20% grade reduction per late day submission
There are 8 problems to this homework

1. Read the book, Chapter 4, *except* Section 4.4.

2. *Problem 4.1.* If X is a random variable that is uniformly distributed between -1 and 1, find the PDF of $\sqrt{|X|}$ and the PDF of -ln$|X|$.

3. Let X and Y be random variables with means μ_X and μ_Y, variances σ_X^2 and σ_Y^2, respectively, and correlation coefficient ρ.

(a) Calculate the value of the constant a such that $E[(Y - aX)^2]$ is minimized. Express your answer in terms of μ_X, μ_Y, σ_X, σ_Y, and ρ.

(b) Calculate the value of $E[(Y - aX)^2]$ when a is equal to what you calculated in part (a). Express your answer in terms of μ_X, μ_Y, σ_X, σ_Y, and ρ.

4. *Problem 4.4.* The metro train arrives at the station near your home every quarter hour starting at 6:00 a.m. You walk into the station every morning between 7:10 and 7:30 a.m., with the time in this interval being a random variable with given PDF (see example 3.14 in Chapter 3). Let X be the elapsed time, in minutes, between 7:10 and the time of your arrival. Let Y be the time that you have to wait until you board a train. Calculate the CDF of Y in terms of the CDF of X and differentiate to obtain a formula for the PDF of Y.

5. *Problem 4.10.* Let X and Y be independent random variables with PMFs

$$p_X(x) = \begin{cases} 1/3, & \text{if } 1 < x = 1,2,3, \\ 0, & \text{otherwise,} \end{cases}$$

and

$$p_Y(y) = \begin{cases} 1/2, & \text{if } 1 < y = 0, \\ 1/3, & \text{if } 1 < y = 1, \\ 1/6, & \text{if } 1 < y = 2, \\ 0, & \text{otherwise,} \end{cases}$$

Find the PMF of $Z = X + Y$ using the convolution formula.

6. *Problem 4.17.* Suppose that X and Y are random variables with the same variance. Show that $X - Y$ and $X + Y$ are uncorrelated.

7. *Problem 4.23.* Pat and Nat are dating, and all of their dates are scheduled to start at 9:00 PM. Nat always arrives promptly at 9:00 PM. Pat is highly disorganized and arrives at a time that is uniformly distributed between 8:oo PM and 10:00 PM. Let X be the time in hours between 8:00 PM and the time when Pat arrives. If Pat arrives before 9:00 PM, their date will last exactly 3 hours. If Pat arrives after 9:00 PM, their date will last for a time that is uniformly distributed between 0 and 3 - X hours. The date starts at the time they meet. Nat gets irritated when Pat is late and will end the relationship after the second date on which Pat is late by more than 45 minutes. All dates are independent of any other dates. *Hint: Define a random variable W = the number of hours that Pat waits.*

(a) What is the expected number of hours Nat waits for Pat to arrive?

(b) What is the expected duration of any particular date?

(c) What is the expected number of dates they will have before breaking up?

8. Suppose that X and Y are independent, and both have exponential continuous distributions:

$$f_X(x) = ae^{-ax}, \text{ for } x \geq 0,$$

and

$$f_Y(y) = be^{-by}, \text{ for } y \geq 0,$$

where a and b are constants ($a \neq b$). Calculate the PDF of $Z = X + Y$ by performing the convolution integral. (*Note:* This problem is more easily solved if you make use of the step function. The step function $u(x)$ is defined such that it is equal to 1 if $x \geq 0$) and it is equal to 0 if $x < 0$. This means that we can write $f_X(x)$ as $f_X(x) = ae^{-ax}u(x)$. The same applies to $f_Y(y) = be^{-by}u(y)$. You can then easily use f_X and f_Y as rewritten with the step functions in the integral and by applying their effect to the limits of integration, the step functions greatly simplify calculations.)