

**08 Oct. 2014 – Due MONDAY, 20 Oct. 2014 in Dropbox**  
**20% grade reduction per late day submission**  
**There are 11 problems to this homework**

1. Read the book, all of Chapter 3.
2. Calculate the variance in Example 3.4 with the full equations for the definition of the variance, *i.e.*,  $\int_{-\infty}^{\infty} (x - E[X])^2 f_X(x) dx$ .
3. Read the section on the Exponential Random Variable (you are supposed to anyway because of problem 1 ☹). Calculate here again the variance of  $f_X(x)$  using the definition  $\int_{-\infty}^{\infty} (x - E[X])^2 f_X(x) dx$ .

4. *Problem 3.2 - Laplace Random Variable.* Let  $X$  have the PDF

$$f_X(x) = \frac{\lambda}{2} e^{-\lambda|x|},$$

where  $\lambda$  is a positive scalar. Verify that  $f_X(x)$  satisfies the normalization condition, and evaluate the mean and variance of  $X$ .

5. A random variable has a CDF given by  $F_X(x) = (1 - e^{-x})$ , for  $x \geq 0$ . Calculate the following:
  - (a)  $P(X > 3)$
  - (b)  $P(X < 5 \mid X > 3)$
  - (c)  $P(X > 6 \mid X > 3)$
  - (d)  $P((|X - 5| < 4) \mid (|X - 6| > 2))$

6. *Problem 3.5.* Consider a triangle and a point chosen within the triangle according to the uniform probability law. Let  $X$  be the distance from the point to the base of the triangle. Given the height of the triangle, find the CDF and the PDF of  $X$ .

7. *Problem 3.7.* Alvin throws darts at a circular target of radius  $r$  and is equally likely to hit any point in the target. Let  $X$  be the distance of Alvin's hit from the center.
  - (a) Find the PDF, the mean, and the variance of  $X$ .
  - (b) The target has an inner circle of radius  $t$ . If  $X \leq t$ , Alvin gets a score of  $S = 1/X$ . Otherwise his score is  $S = 0$ . Find the CDF of  $S$ . Is  $S$  a continuous random variable?

8. *Problem 3.11.* Let  $X$  and  $Y$  be normal random variables with means 0 and 1, respectively, and variances 1 and 4, respectively. Use the table on page 155 of the book, when needed, to calculate:
  - (a)  $P(X \leq 1.5)$  and  $P(X \leq -1)$ .
  - (b) The PDF of  $(Y - 1)/2$ .
  - (c)  $P(-1 \leq Y \leq 1)$ .

9. *Problem 3.18.* Let  $X$  be a random variable with PDF

$$f_X(x) = \begin{cases} x/4, & \text{if } 1 < x \leq 3, \\ 0, & \text{otherwise,} \end{cases}$$

and let  $A$  be the event  $\{X \geq 2\}$ .

- (a) Find  $E[X]$ ,  $P(A)$ ,  $f_{X|A}(x)$ , and  $E[X|A]$ .
- (b) Let  $Y = X^2$ . Find  $E[Y]$  and  $\text{var}(Y)$ .

**10.** *Problem 3.20* An absent-minded professor schedules two student appointments for the same time. The appointment durations are independent and exponentially distributed with mean thirty minutes. The first student arrives on time, but the second student arrives five minutes late. What is the expected value of the time between the arrival of the first student and the departure of the second student?

**11.** *Problem 3.34.* A defective coin minting machine produces coins whose probability of heads is a random variable  $P$  with PDF

$$f_P(p) = \begin{cases} pe^p, & p \in [0, 1], \\ 0, & \text{otherwise.} \end{cases}$$

A coin produced by this machine is selected and tossed repeatedly, with successive tosses assumed independent.

- (a) Find the probability that a coin toss results in heads.
- (b) Given that a coin toss resulted in heads, find the conditional PDF of  $P$ .
- (c) Given that the first coin toss resulted in heads, find the conditional probability of heads on the next toss.