24 Sept. 2014 – Due MONDAY, 29 Sept. 2014 in Dropbox
20% grade reduction per late day submission
There are 9 problems to this homework

1. Read the book, all of Chapter 2.

2. *Problem 2.2.* You go to a party with 500 guests. What is the probability that exactly one other guest has the same birthday as you? Calculate this exactly and also approximately by using the Poisson PMF. (For simplicity, exclude birthdays on February 29.)

3. *Problem 2.16.* Let X be a random variable with PMF:

$$p_X(x) = \begin{cases} x^2/a, & \text{if } x\text{= -3, -2, -1, 0, 1, 2, 3,} \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find a and E[X].
(b) What is the PMF of the random variable $Z = (X - E[X])^2$?
(c) Using the result from (b), find the variance of X.
(d) Find the variance of X using the formula var(X)=$\sum_x (x - E[X])^2 p_X(x)$.

4. *Problem 2.24.* A stock market trader buys 100 shares of stock A and 200 shares of stock B. Let X and Y be the price changes of A and B, respectively, over a certain time period, and assume that the joint PMF of X and Y is uniform over the set of integers x and y satisfying:

$$-2 \le x \le 4, \ -1 \le y - x \le 1.$$

(a) Find the marginal PMF's and the means of X and Y.
(b) Find the mean of the trader's profit.

5. *Problem 2.36 - Multiplication rule for conditional PMFs.* Let X, Y and Z be random variables.

(a) Show that

$$p_{X,Y,X}(x, y, z) = p_X(x)p_{Y|X}(y|x)p_{Z|X,Y}(z|x, y).$$

(b) How can you interpret this formula as a special case of the multiplication rule that we saw in class (see section 1.3 in the book)?
(c) Generalize to the case of more than 3 random variables.

6. *Problem 2.38.* Alice passes through four traffic lights on her way to work, and each light is equally likely to be green or red. Each light is independent of the other.

(a) What is the PMF, the mean, and the variance of the number of red lights that Alice encounters?
(b) Suppose that each red light delays Alice by exactly two minutes. What is the variance of Alice's commuting time?

7. *Problem 2.41.* You drive to work 5 days a week for a full year (50 weeks), and with probability $p = 0.02$ you get a traffic ticket on any given day, independent of other days. Let X be the total number of tickets you get in the year.

(a) What is the probability that the number of tickets you get is exactly equal to the expected value of X?
(b) Calculate approximately the probability in (a) using a Poisson approximation.
(c) Any one of the tickets is \$10 or \$20 or \$50 with respective probabilities 0.5, 0.3, and 0.2, and independent of other tickets. Find the mean and the variance of the amount of money that you pay in traffic tickets during the year.

(d) Suppose that you don't know the probability p of getting a ticket, but you got 5 tickets during the year, and you estimate p by the sample mean

$$\hat{p} = \frac{5}{250} = .02.$$

What is the range of possible values of p assuming that the difference between p and the sample mean $\hat{p}$ is within 5 times the standard deviation of the sample mean?

8. *Problem 2.44.* Let X and Y be two random variables with given joint PMF, and let g and h be two functions of X and Y, respectively. Show that if X and Y are independent, then the same is true for the random variables $g(X)$ and $h(Y)$.

9. Imagine a circular room, and that you are trapped in it. This room has 3 identical doors that are symmetrically placed around the perimeter of the room. Some voice coming through into the room tells you that one of the doors leads to the outside after a trip of 2 hours through a maze. But, the other two doors also lead to mazes, but these mazes end up back into the room after a 2 hour trip. To make things worse, when you come back into the room from leaving the room through one of the wrong doors, you are unable to tell through which door you exited or entered. What is the average time for escape to outside of the room? Can you guess the answer ahead of time? If not, explain a physical explanation for the answer that you get.