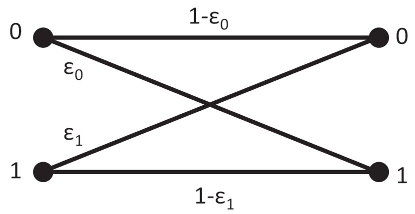
17 Sept. 2014 – Due Wed., 24 Sept. 2014 in Dropbox
20% grade reduction per late day submission
There are 9 problems to this homework

1. Read the book: finish Chapter 1, and read Chapter 2 (pages 72-118).

2. *Problem 1.31 - Communication in a noisy channel.* A source transmits a message (string of symbols) through a noisy communication channel. Each symbol is a 0 or 1 with probability p and $1-p$, respectively, and is received incorrectly with probability ϵ_0 and ϵ_1, respectively (see figure below). Errors in different symbol transmissions are independent.



(a) What is the probability that the k^{th} symbol is received correctly?
(b) What is the probability that the string of symbols 1011 is received correctly?
(c) In an effort to improve reliability, each symbol is transmitted three times and the received string is decoded by majority rule. In other words, a 0 (or 1) is transmitted as 000 (or 111, respectively), and it is decoded at the receiver as a 0 (or 1) *iff* the received three-symbol string contains at leats two 0's (or 1's, respectively). What is the probability that a 0 is correctly decoded?
(d) For what values of ϵ_0 is there an improvement in the probability of correct decoding of a 0 when the scheme of part (c) is used?
(e) Suppose that the scheme of part (c) is used. What is the probability that a symbol was a 0 given that the received string is 101?

3. A poker hand consists of 5 cards that are drawn from a regular deck of cards (52 cards).

(a) How many different poker hands are there?
(b) How many different poker hands that contain all four aces are there? What is the probability of being dealt all four aces in one hand? What is the probability that you are dealt four-of-a-kind (*i.e.*, four aces, or four queens, or four jacks, etc.)?
(c) How many different poker hands are there that contain exactly three aces? What is the probability of being dealt a hand that contains three aces? Lastly, what is the probability of being dealt three-of-a-kind?

4. Two events A and B are independent.

(a) Is A independent of B^c? If yes, prove it, and if no, give a counter example.
(b) Is A^c independent of B^c? If yes, prove it, and if no, give a counter-example.

5. *Problem 1.37.* A cellular phone system services a population of n_1 "voice users" (those who occasionally need a voice connection) and n_2 "data users" (those who occasionally need a data connection). We estimate that at a given time, each user will need to be connected to the system with probability p_1 (for voice users) or p_2 (for data users), independent of other users. The data rate for a voice user is r_1 bits/sec and for a data user is r_2 bits/sec. The cellular system has a total capacity of c bits/sec. What is the probability that more users want to use the system than the system can accommodate?

6. *Modified Problem 1.49 - De Méré's puzzle.* A six-sided die is rolled three times independently. Which is more likely: a sum of 11 or a sum of 12? (This question was posed by the French nobleman de Méré to his friend Pascal in the 17th century.) Now, of the following three, which is more likely: a sum of 10, 11 or 12?

7. *Problem 1.53.* Ninety students, including Joe and Jane, are to be split into three classes of equal size, and this is to be done at random. What is the probability that Joe and Jane end up in the same class?

8. *Problem 1.62 - Correcting the number of permutations for indistinguishable objects.* When permuting n objects, some of which are indistinguishable, different permutations may lead to indistinguishable object sequences, so the number of distinguishable object sequences is less than $n!$. For example, there are six permutations of the letters A, B, and C:

$$ABC, ACB, BAC, BCA, CAB, CBA,$$

but only three distinguishable sequences that can be formed using the letters A, D, and D:

$$ADD, DAD, DDA.$$

(a) Suppose that k out of the n objects are indistinguishable. Show that the number of distinguishable object sequences is $n!/k!$.

(b) Suppose that we have r types of indistinguishable objects, and for each i, we have k_i objects of type i. Show that the number of distinguishable object sequences is

$$\frac{n!}{k_1!k_2!...k_r!}.$$

9. An audio amplifier made up of six transistors is not working well. You determine that two transistors are defective but you do not know which ones. You remove three transistors at random and inspect them. Let X be the number of defective transistors that you find, where X may be 0, 1 or 2. Find the PMF for x.