

1. OK!

2. Let G and C be the events that the chosen student is a genius and a chocolate lover, respectively. We have $P(G) = 0.6$, $P(C) = 0.7$, and $P(G \cap C) = 0.4$. We are interested in $P(G^c \cap C^c)$, which is obtained with the following calculation:

$$P(G^c \cap C^c) = 1 - P(G \cup C) = 1 - (P(G) + P(C) - P(G \cap C)) = 1 - (0.6 + 0.7 - 0.4) = 0.1.$$

3. Let p_i be the probability of winning against the opponent played in the i^{th} turn. Then, you will win the tournament if you win against the 2^{nd} player (probability p_2) and also you win against at least one of the two other players (probability $p_1 + (1 - p_1)p_3 = p_1 + p_3 - p_1p_3$). Thus, the probability of winning the tournament is

$$p_2(p_1 + p_3 - p_1p_3).$$

The order (1, 2, 3) is optimal if and only if the above probability is no less than the probabilities corresponding to the two alternative orders, *i.e.*,

$$p_2(p_1 + p_3 - p_1p_3) \geq p_1(p_2 + p_3 - p_2p_3),$$

$$p_2(p_1 + p_3 - p_1p_3) \geq p_3(p_2 + p_1 - p_2p_1),$$

The first inequality above is equivalent to $p_2 \geq p_1$, while the second inequality above is equivalent to $p_2 \geq p_3$.

4.

(a) Each possible outcome has probability $\frac{1}{36}$. There are 6 possible outcomes that are doubles, so the probability of doubles is $\frac{6}{36} = \frac{1}{6}$.

(b) The conditioning event (sum is 4 or less) consists of the 6 outcomes

$$\{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (3, 1)\},$$

from which 2 are doubles, so the conditional probability of doubles is $\frac{2}{6} = \frac{1}{3}$.

(c) There are 11 possible outcomes with at least one 6, namely, (6, 6), (6, i), and (i , 6), for $i = 1, 2, \dots, 5$. Thus, the probability that at least one die is a 6 is $\frac{11}{36}$.

5. Write solution to this problem

6.

(a) $\frac{6}{36} = \frac{1}{6}$.

(b) Using this MATLAB code:

```
N = [1000 2000 4000 10000 100000 1000000];
for j = 1 : length(N)
    die1 = ceil( 6*rand(1,N(j))); \% Die 1 is tossed N times
    die2 = ceil( 6*rand(1,N(j))); \% Die 2 is tossed N times
    sumDice = die1 + die2; \% Sum of both tosses
    numberOfSumsEqualTo7 = sum( sumDice \text{==} 7 ); \% Add up the # times the sum=7
    occurrenceOf7(j) = numberOfSumsEqualTo7 / N(j);
end
```

you can plot the results using, for example, `semilogx(N, occurrenceOf7, '*')`, and you get something as plotted on the next page. As the number of tosses increases, we get closer and closer to the theoretical limit of what the probability actually is. So to obtain the probability that we calculate on paper, we need to perform the experiment an infinite number of times.

