ECE 303: Homework #9 Solutions

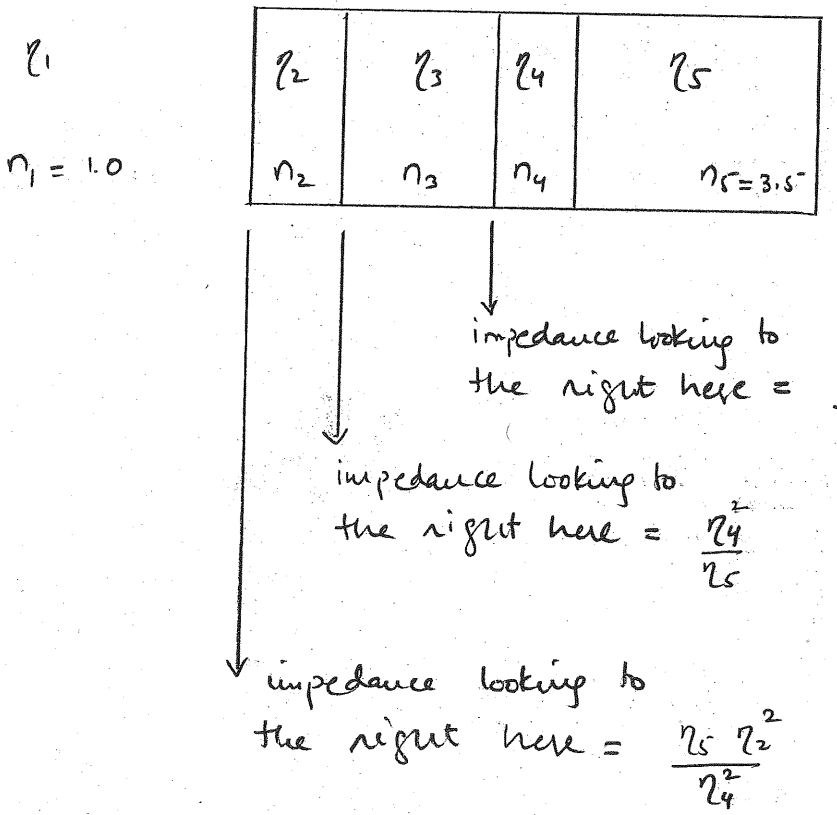
(By Farhan Rana)

9.1

a) $n_2 = \sqrt{n_1 n_3} = \sqrt{3.5}$ thickness $= \frac{\lambda_2}{4} = \frac{0.52/\sqrt{3.5}}{4} = 69.5 \text{ nm}$
$= 69.5 \times 10^{-9} \text{ m}$

b) See attached plot and routine

c) Work with impedances: $\eta = \frac{\eta_0}{n}$



Need $\frac{\eta_5 \eta_2^2}{\eta_4^2} = \eta_1$ for $|\Gamma|^2 = 0$ $\Rightarrow$ $\left(\frac{\eta_2}{\eta_4}\right)^2 = \frac{\eta_1}{\eta_5}$

$\Rightarrow$ $\left(\frac{n_4}{n_2}\right)^2 = \frac{n_5}{n_1} = 3.5$ $\Rightarrow$ Choose $n_4 = 2.8$ and $n_2 = 1.5$.

Notice that the half-wavelength long layer does nothing. We can even get rid of it without changing anything.

Thickness of 2nd layer $= \frac{0.52/1.5}{4} = 86.7 \text{ nm}$

_____ 3rd layer $= 0$

_____ 4th layer $= \frac{0.52/2.8}{4} = 46.4 \text{ nm}$

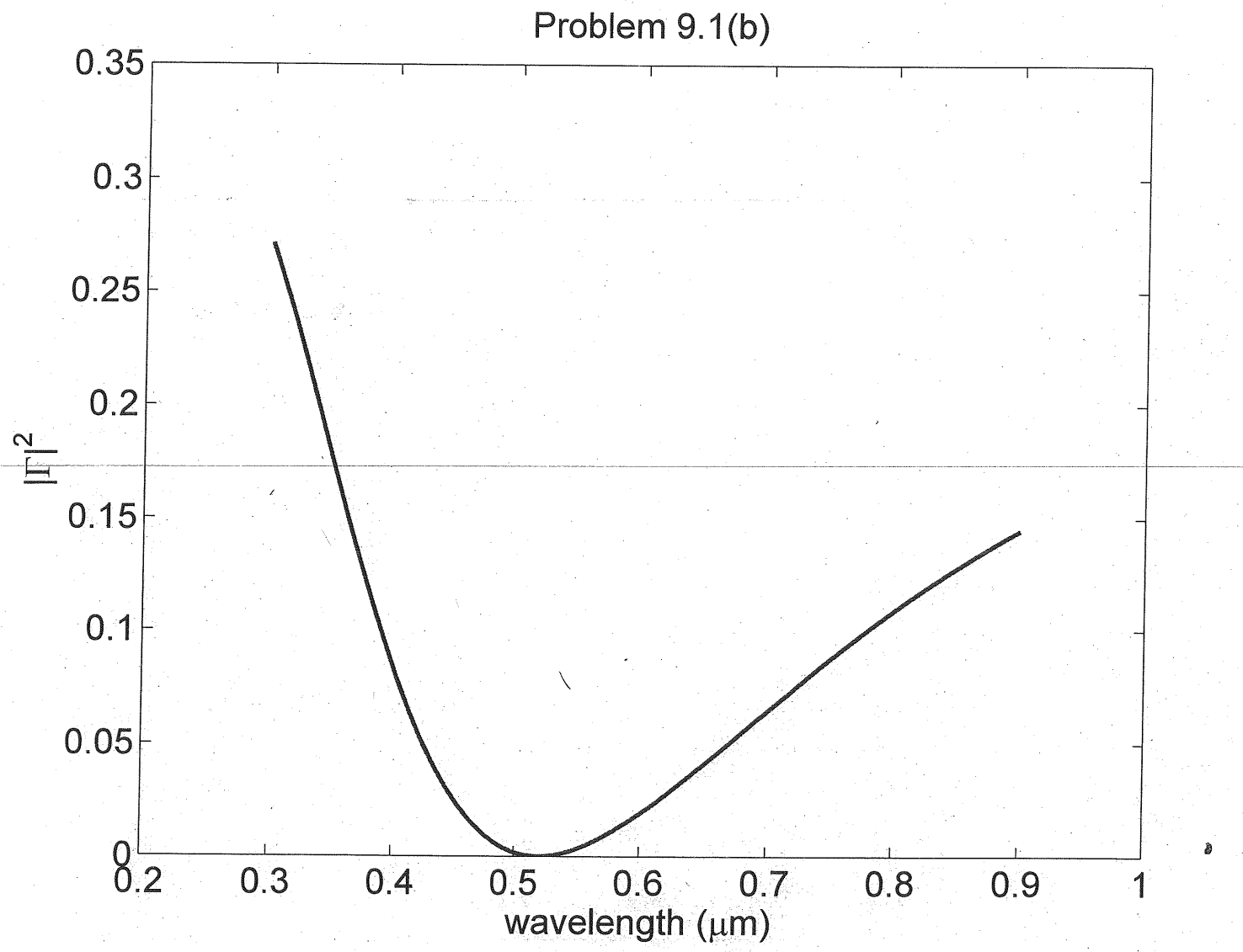
d) See attached plot and routine.

9.2

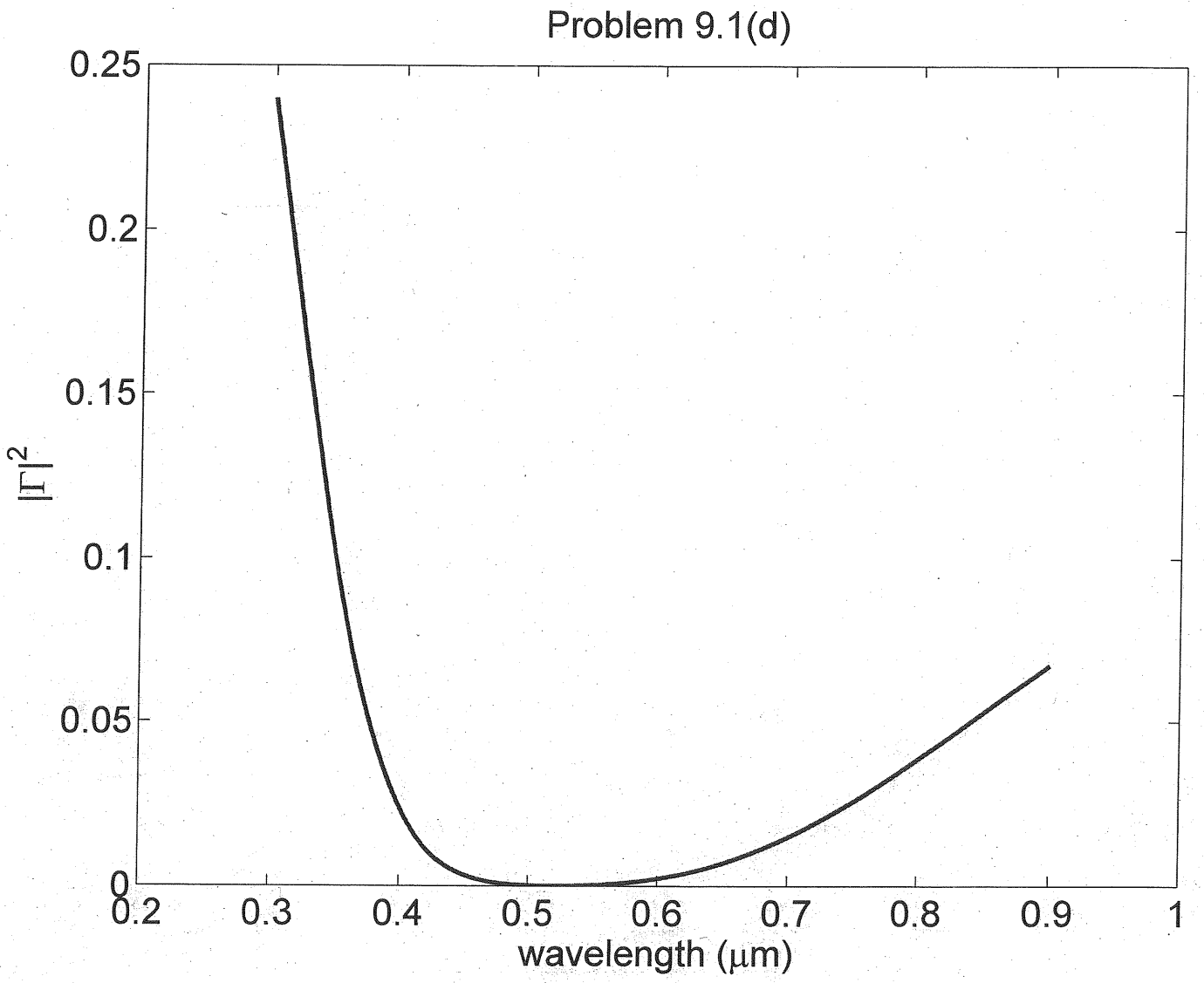
a) Thickness of layer with index $= n_2 = 2.4$ is $\frac{0.52/2.4}{4} = 54.2$ nm

Thickness of layer with index $= n_3 = 1.5$ is $\frac{0.52/1.5}{4} = 86.7$ nm.

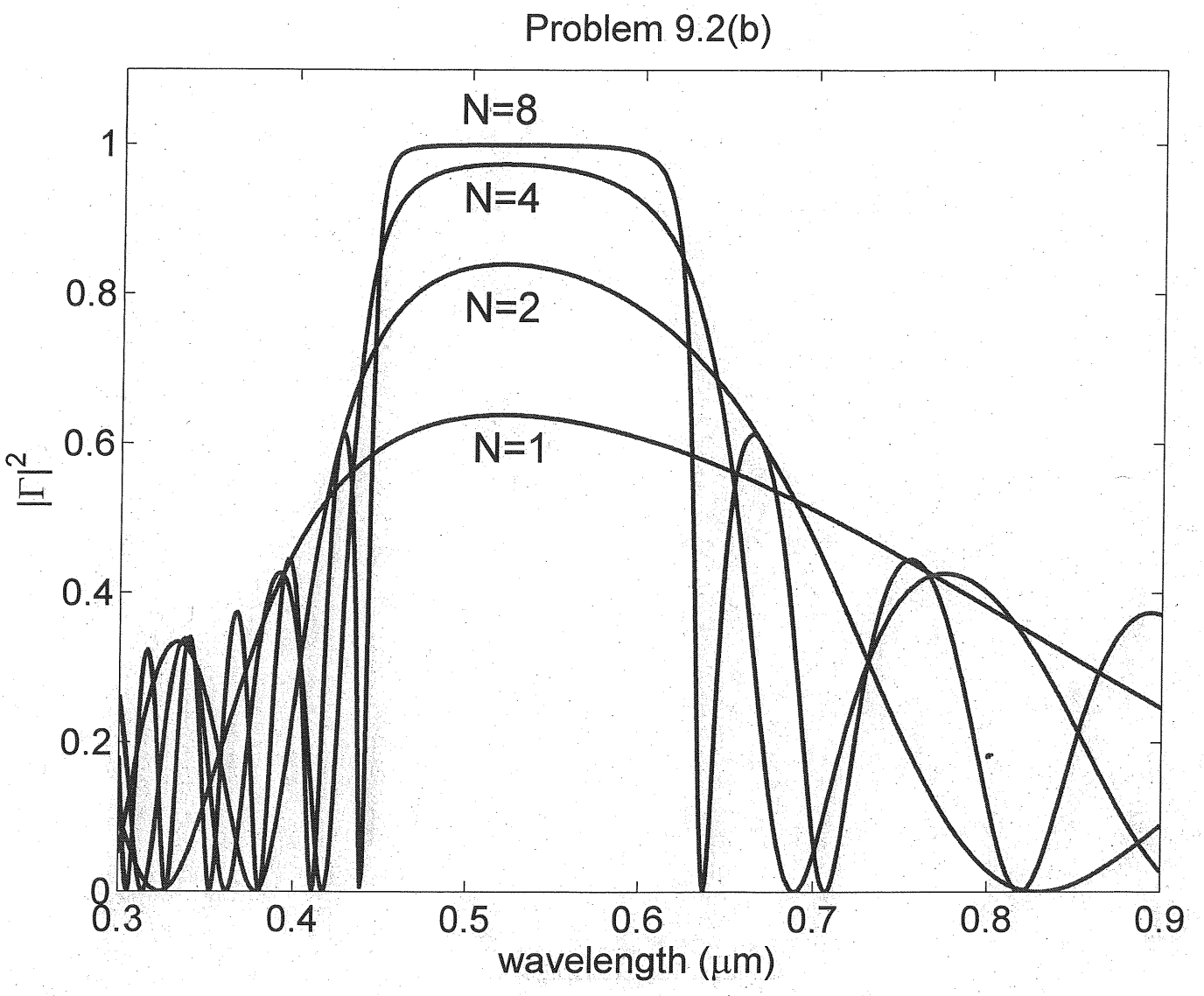
b) See attached sheet and plot.

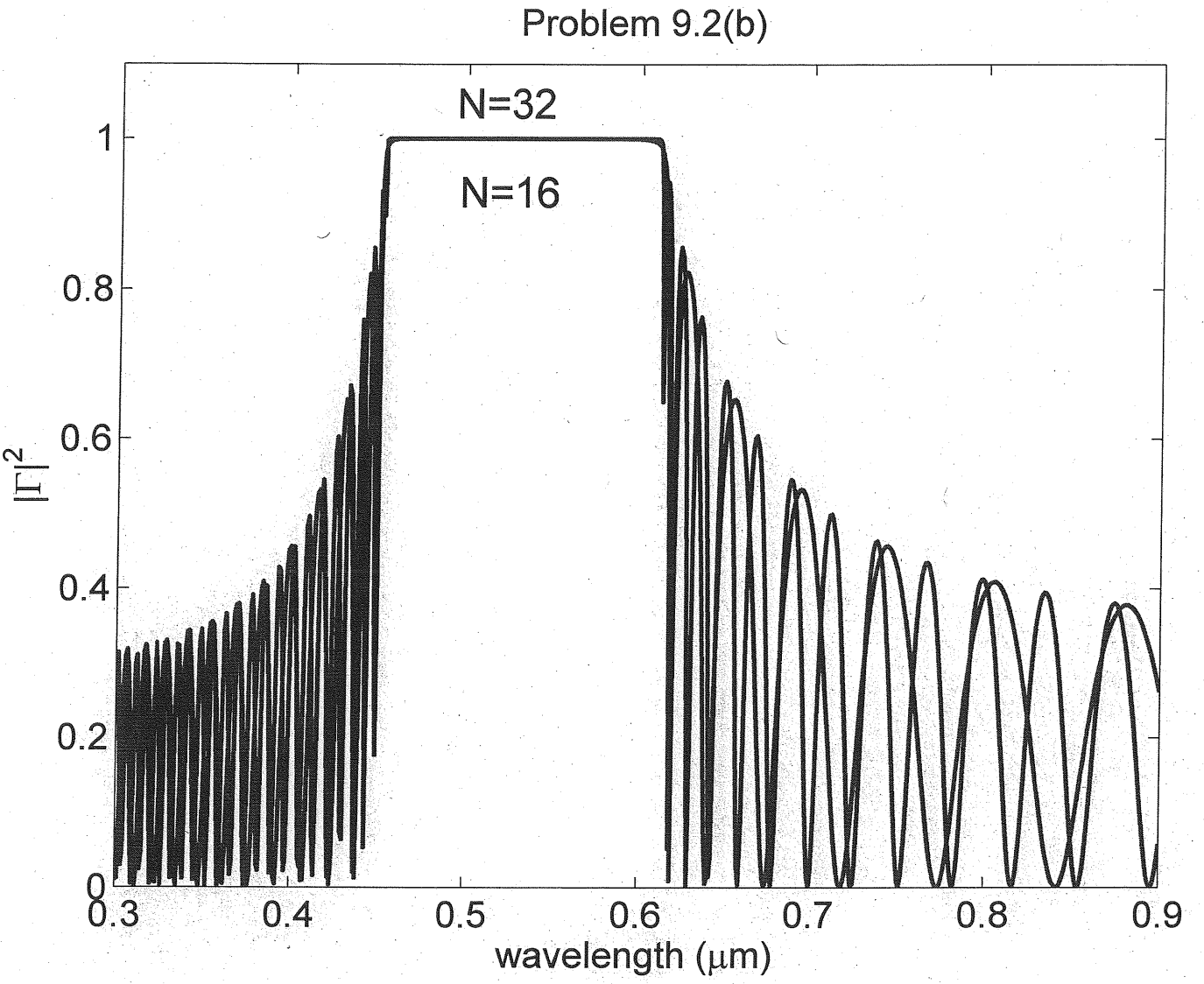

Problem 9.1(b)
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
$|\Gamma|^2$
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
wavelength (μm)

```
% PROBLEM 9.1(b)

%constants
c = 3e8;
muo = 4*pi*1e-7;
epsilono = 8.85e-12;
eatao = sqrt(muo/epsilono);

%wavelength 1D array
lambda = [0.3:.001:0.9]*1e-6;

%structure
n2 = sqrt(3.5);
eata2 = eatao/n2;
l2 = 0.52e-6/(4*n2); %layer thickness
k2 = 2*pi*n2./lambda; %layer wavevector 1D array

n3 = 3.5;
eata3 = eatao/n3

%actual computation using matlabs array processing features
Gamma = (eata3/eata2 - 1)/(eata3/eata2 + 1);
eata = eata2*( 1 + Gamma*exp(-2*j*k2*l2) )./( 1 - Gamma*exp(-2*j*k2*l2) );
Gamma = (eata/eatao - 1)./(eata/eatao + 1);

%plot results
plot(lambda/1e-6,abs(Gamma).^2);

%labels
xlabel('wavelength (\mum)');
ylabel('|\Gamma|^{2}');
title('Problem 9.1(b)');
```


Problem 9.1(d)
0.25
0.2
0.15
0.1
0.05
0
$|\Gamma|^2$
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
wavelength (μm)

```
% PROBLEM 9.1(d)

%constants
c = 3e8;
muo = 4*pi*1e-7;
epsilono = 8.85e-12;
eatao = sqrt(muo/epsilono);

%wavelength range
lambda = [0.3:.001:.9]*1e-6; %1D array
omega = 2*pi*c./lambda; %1D array

%structure
n2 = 1.5;
eata2 = eatao/n2;
l2 = 0.52e-6/(4*n2); %layer thickness
k2 = 2*pi*n2./lambda; %layer wavevector 1D array

%NO THIRD LAYER

n4 = 2.8;
eata4 = eatao/n4;
l4 = 0.52e-6/(4*n4); %layer thickness
k4 = 2*pi*n4./lambda; %layer wavevector 1D array


n5 = 3.5;
eata5 = eatao/n5;

%actual computation using matlabs array processing features
% start from the right side and keep repeating the same code for all layers
% till you reach the left side

Gamma = (eata5/eata4 - 1)/(eata5/eata4 + 1);
eata = eata4*( 1 + Gamma*exp(-2*j*k4*l4) )./( 1 - Gamma*exp(-2*j*k4*l4) );
Gamma = (eata/eata2 - 1)./(eata/eata2 + 1);
eata = eata2*( 1 + Gamma.*exp(-2*j*k2*l2) )./( 1 - Gamma.*exp(-2*j*k2*l2) );
Gamma = (eata/eatao - 1)./(eata/eatao + 1);

%plot results
plot(lambda/1e-6,abs(Gamma).^2);

%labels
xlabel('wavelength (\mum)');
ylabel('|\Gamma|^{2}');
title('Problem 9.1(d)');
```


Problem 9.2(b)
N=8
N=4
N=2
N=1
$|\Gamma|^2$
1
0.8
0.6
0.4
0.2
0
0.3
0.4
0.5
0.6
0.7
0.8
0.9
wavelength (μm)


Problem 9.2(b)
N=32
N=16
1
0.8
0.6
0.4
0.2
0
$|\Gamma|^2$
0.3
0.4
0.5
0.6
0.7
0.8
0.9
wavelength (μm)

```
% PROBLEM 9.2(b)

%constants
c = 3e8;
muo = 4*pi*1e-7;
epsilono = 8.85e-12;
eatao = sqrt(muo/epsilono);

%wavelength range
lambda = [0.3:.001:.9]*1e-6; %1D array
omega = 2*pi*c./lambda; %1D array

%structure

N=8; %number of pairs

n2 = 2.4;
eata2 = eatao/n2;
l2 = 0.52e-6/(4*n2); %layer thickness
k2 = 2*pi*n2./lambda; %layer wavevector 1D array

n3 = 1.5;
eata3 = eatao/n3;
l3 = 0.52e-6/(4*n3); %layer thickness
k3 = 2*pi*n3./lambda; %layer wavevector 1D array


n4 = 3.5;
eata4 = eatao/n4;

%actual computation using matlabs array processing features
% start from the right side and keep repeating the same code for all layers
% till you reach the left side

eata = eata4 %starting mpedance

for n=1:1:N

    Gamma = (eata/eata3 - 1)./(eata/eata3 + 1);
    eata = eata3*( 1 + Gamma.*exp(-2*j*k3*l3) )./( 1 - Gamma.*exp(-2*j*k3*l3) );
    Gamma = (eata/eata2 - 1)./(eata/eata2 + 1);
    eata = eata2*( 1 + Gamma.*exp(-2*j*k2*l2) )./( 1 - Gamma.*exp(-2*j*k2*l2) );
end

Gamma = (eata/eatao - 1)./(eata/eatao + 1);

%plot results
plot(lambda/1e-6,abs(Gamma).^2);

%labels
xlabel('wavelength (\mum)');
ylabel('|\Gamma|^{2}');
title('Problem 9.2(b)');
```