ECE 303: Electromagnetic Fields and Waves

Fall 2007

Homework 8 **Due on Oct. 19, 2007 by 5:00 PM**

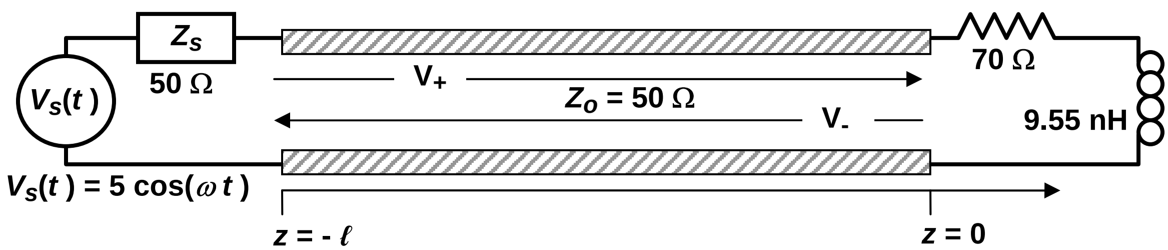
Reading Assignments:

i) Review the lecture notes.
ii) Review sections 4.1-4.3, 5.1-5.2, 5.4, 6.1, 6.3-6.4, paperback book *Electromagnetic Waves*. These sections also include the material to be covered in the next two weeks of the class.

Special Note: Graders have been instructed to take off points (as much as 50%) if proper units are not included in your numerical answers. You must specify the correct units with your numerical answers.

Problem 8.1: (Impedance transformations in microwave circuits)

Consider the following transmission line circuit:



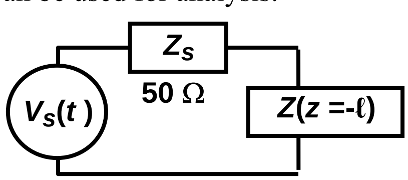
The circuit is operating at a frequency of 1.0 GHz. At the load end of the circuit there is a 70 Ω resistor in series with a 9.55 nH = 9.55×10^{-9} H inductor. The length of the transmission line is such that:

$$\ell = \frac{21}{16}\lambda$$

where λ is the wavelength of waves in the transmission line at a frequency of 1.0 GHz.

a) Find the load reflection coefficient Γ_L (give a numerical value).

b) Find the impedance $Z(z = -\ell)$ looking into the transmission line at $z = -\ell$ so that the following equivalent circuit can be used for analysis:



Give a numerical value for your answer.

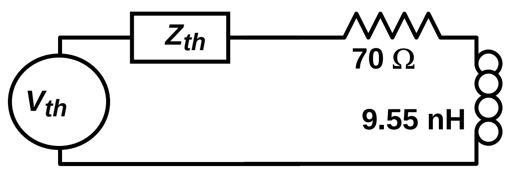
c) Find the time-average power dissipated in the impedance $Z(z = -\ell)$ in the above circuit (give a numerical value).

d) Find the voltage V_+ of the forward going voltage wave in the transmission line circuit (give a numerical value).

e) Find the voltage V_- of the backward going voltage wave in the transmission line circuit (give a numerical value).

f) Find the net time-average power traveling in the +z-direction in the transmission line circuit (give a numerical value). Compare your answer with your answer in part (c) and explain what you learned by this comparison.

g) Now find the Thevenin equivalent of the "source+transmission line" so that the following equivalent circuit can be used for analysis:

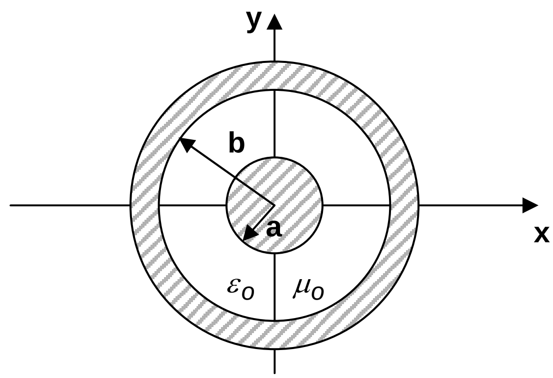


You need to find the impedance Z_{th} and the voltage phasor V_{th} (give numerical answers for each of these).

h) Using your circuit of part (g) find the time-average power dissipated in the 70 Ω load resistor (give a numerical value). Compare your answer to your answers in parts (c) and (f) and explain what you learned by this comparison.

Problem 8.2: (Energy flow and power in transmission lines)

Consider a co-axial transmission line whose cross-section is shown below:



Suppose a voltage-current wave given by:

$$V(z) = V_+ \, e^{-jk\,z}$$

$$I(z) = I_+ \, e^{-jk\,z} = \frac{V_+}{Z_o} e^{-jk\,z}$$

is traveling in the co-axial line. In the lecture notes you were told that the time-average power is related to the Poynting vector through the relation:

$$\langle P_z(t)\rangle = \iint \langle \vec{S}(\vec{r},t)\rangle . \hat{z}\, dx\, dy = \frac{1}{2}\iint \mathrm{Re}\left[\vec{S}(\vec{r})\right] . \hat{z}\, dx\, dy = \frac{1}{2}\mathrm{Re}\left[V_+ \, I_+^*\right]$$

In this problem you are going to prove the last equality of the above relation for a co-axial line.

a) Find the expression for the impedance Z_o of the co-axial line in terms of the dimensions specified in the figure above. You can write down the answer using lecture notes – no need to compute from first principles.

b) If the voltage wave is given by $V(z)=V_+ \, e^{-jk\,z}$, find an expression for the position dependent electric field vector phasor associated with the wave in the annular region between the two conductors.

Hint: You can assume that a positive value of the voltage implies that the center conductor is at a higher potential. $\vec{E}(\vec{r})=?$

c) If the current wave is given by $I(z)=I_+ \, e^{-jk\,z}$ find an expression for the magnetic field vector phasor associated with the wave in the annular region between the two conductors.

Hint: You can assume that a positive value of the current implies that the current in the center conductor is in the +z-direction. $\vec{H}(\vec{r})=?$

d) Using your results from parts (b) and (c), find the complex Poynting vector $\vec{S}(\vec{r})$ in the annular region between the two conductors. Which way is the power flowing?

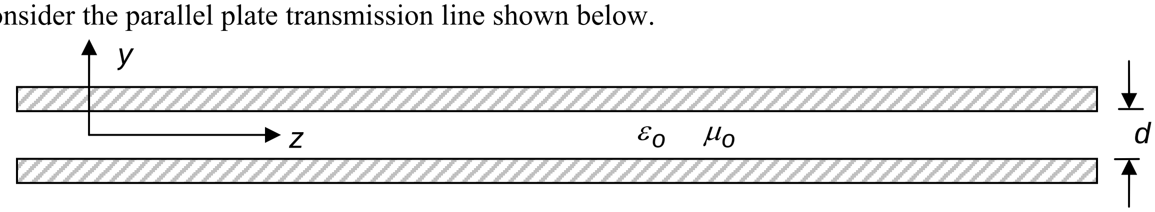
e) Integrate the complex Poynting vector $\vec{S}(\vec{r})$ obtained in part (d) above over the cross-sectional area of the annular region between the two conductors and show that:

$$\frac{1}{2}\iint \mathrm{Re}\left[\vec{S}(\vec{r})\right].\hat{z}\,dx\,dy=\frac{1}{2}\mathrm{Re}\left[V_+\,I_+^*\right]$$

Problem 8.3: (Lossy Transmission Lines)

Perfect metals and dielectrics are generally not available in this world to make transmission lines. Consequently, real transmission lines are lossy (i.e. a wave propagating in a transmission line looses power as it propagates). One contributing factor towards this loss is the I^2R dissipation in the imperfect metals of the transmission line. Any real metal will have some finite resistance and current flow will result in power dissipation. In this problem you will consider a more realistic model of a transmission line.

Consider the parallel plate transmission line shown below.



The width of the metal plates is W (in the x-direction). The capacitance and inductance **per unit length** of the transmission line are C and L, respectively. Suppose the total combined resistance **per unit length** of the top and bottom plates is R.

a) Show that when resistance is present the telegrapher's equations become,

$$\frac{\partial V(z,t)}{\partial z} = -L\frac{\partial I(z,t)}{\partial t} - R\,I(z,t)$$

$$\frac{\partial I(z,t)}{\partial z} = -C\frac{\partial V(z,t)}{\partial t}$$

Hint: Use the same methods as discussed in the lecture notes to derive the above equations.

b) Convert the above equations into phasor notation, and then derive the complex wave equation for the voltage phasor $V(z)$.

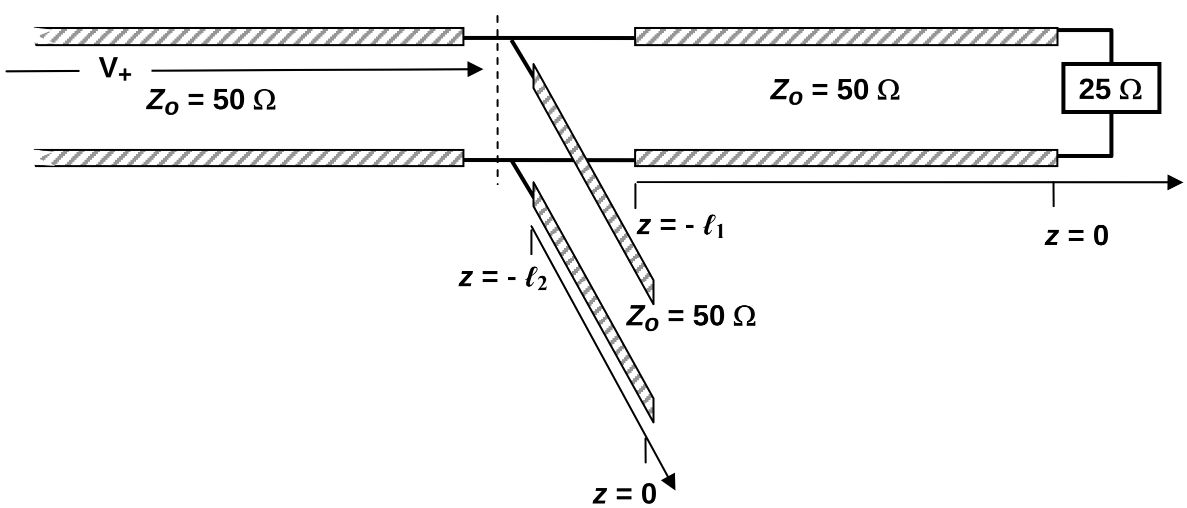
c) Assume a propagating solution of the form $V(z) = V_+\, e^{-jk\,z}$, plug it into the wave equation you derived in part (b), and find the k-vs-ω dispersion relation for the lossy transmission line. Check that your result gives the correct k-vs-ω dispersion relation in the case when R=0.

If you did everything correct to this point you will discover that the k-vector has real and imaginary parts and the imaginary part describes wave decay due to power loss in the signal propagating in the transmission line as a result of the I^2R dissipation in the resistance associated with the metal plates.

d) Find the characteristic impedance Z_o of the transmission line. **Hint:** it will be complex.

Problem 8.4: (Stub tuning in microwave circuits)

Consider the following transmission line circuit shown in the figure below. All transmission lines have an impedance of 50 Ω. On the left is a transmission line carrying an input signal specified by the amplitude V_+ of the forward going voltage wave on that transmission line. On the right is a load impedance of 25 Ω. The goal is to transfer all the input power to the 25 Ω load impedance. Since the load is not matched to the transmission line impedance of 50 Ω, if the load were to be directly connected to the input transmission line then some input power will get reflected back. So we use the circuit shown below.



An open-circuit stub tuner is used to match the total impedance of the structure on the right (of the dashed line) to the 50 Ω impedance of the transmission line on the left carrying the input signal. You have at your disposal two design parameters - you can choose the lengths ℓ_1 and ℓ_2 of both the transmission lines.

Assuming that the wavelength of the waves at the frequency of operation is λ in all the transmission lines, you need to specify the lengths ℓ_1 and ℓ_2 in terms of the wavelength λ such that the impedance of the structure to the right of the dashed line is exactly 50 Ω. **You need to design using Smith Charts.**

Matlab work:

Go to the course website and download the matlab file for the function "smith303.m". The function "smith303" is called as follows:

>> smith303(Z_L, Z_o)

where Z_L and Z_o are the load and transmission line impedances, respectively. In response, smith303 does the following:

i) draws a smith chart

ii) points out the starting point (i.e. $\Gamma(z=0)$ and $Z_n(z=0)$) on the smith chart

iii) draws the circle that shows the values of $\Gamma(z)$ and $Z_n(z)$ on the smith chart as ones moves back from the load on the transmission line.

Solution strategy:

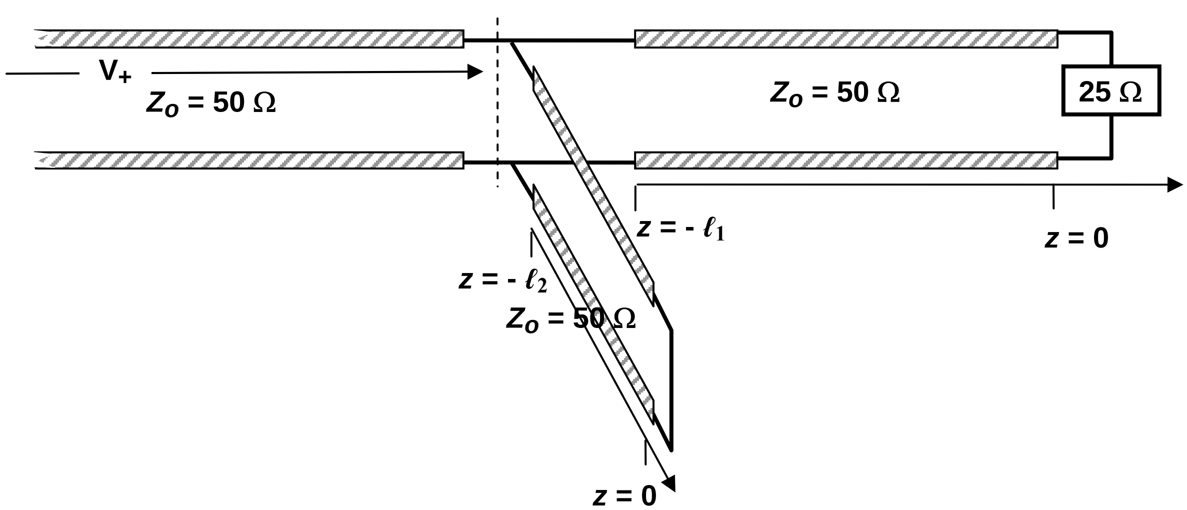
i) First choose the smallest length ℓ_1 such that the normalized **admittance** $Y_n(z=-\ell_1)$ has a real part of unity

ii) Then choose the smallest length ℓ_2 such that the normalized **admittance** $Y_n(z=-\ell_2)$ has an imaginary part that exactly cancels the imaginary part of $Y_n(z=-\ell_1)$.

a) Find the smallest length ℓ_1 in terms of the wavelength λ such that $Y_n(z=-\ell_1)$ has a real part of unity. Use smith chart to calculate ℓ_1 and include a printout of the smith chart showing your work with your answer sheet. Note that the normalized admittance is always diagonally opposite to the normalized impedance on a smith chart.

b) Find the smallest length ℓ_2 in terms of the wavelength λ such that $Y_n(z=-\ell_2)$ has an imaginary part that exactly cancels the imaginary part of $Y_n(z=-\ell_1)$. Use smith chart to calculate ℓ_2 and include a printout of the smith chart showing your work with your answer sheet. Note that the normalized admittance is always diagonally opposite to the normalized impedance on a smith chart.

c) Now suppose that instead of using an open-circuit stub you use a short circuit stub as shown in the figure below.

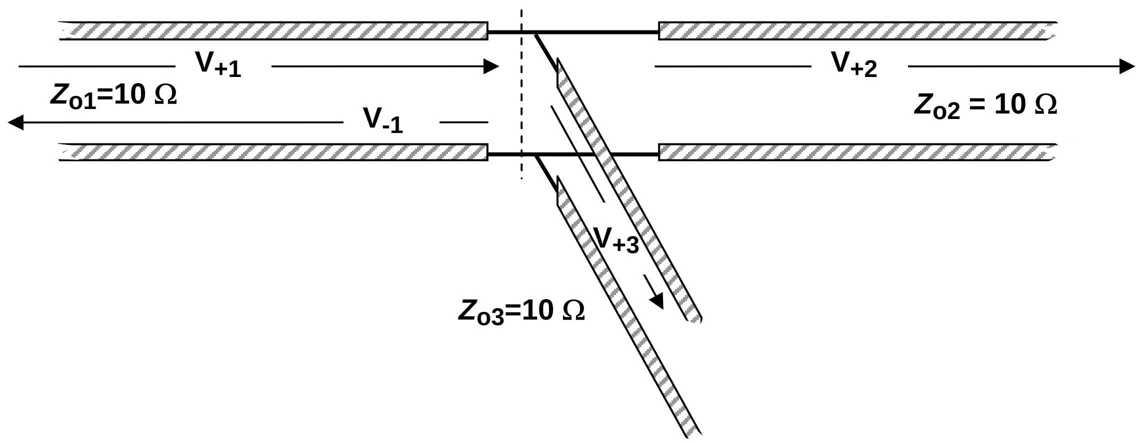


Assuming that the length ℓ_1 is the same as that calculated in part (a) above, find the smallest length ℓ_2 in terms of the wavelength λ such that $Y_n(z=-\ell_2)$ has an imaginary part that exactly cancels the imaginary part of $Y_n(z=-\ell_1)$. Use smith chart to calculate ℓ_2 and include a printout of the smith chart showing your work with your answer sheet. Note that the normalized admittance is always diagonally opposite to the normalized impedance on a smith chart.

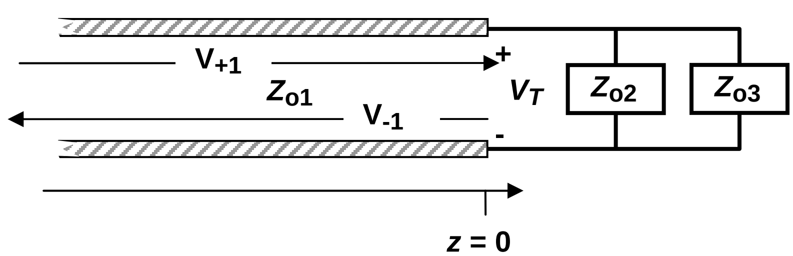
Problem 8.5: (Power splitting in microwave circuits)

Power splitters are commonly used in integrated microwave circuits on a chip to split microwave power into two or more output directions. A schematic of a 1x2 microwave splitter is shown below.

You need to figure out what fraction of the input power is reflected, and what fraction of the input power is transmitted into each of the output transmission lines. Before you can do that you need to find the amplitudes V_{-1} , V_{+2} , and V_{+3} of the voltage waves in terms of the input wave amplitude V_{+1}.



a) Looking to the right of the dashed line, the two output transmission lines can be represented as lumped impedances so the equivalent circuit becomes as shown below:



Find the amplitude V_{-1} of the reflected wave in terms of the input wave amplitude V_{+1} and find the fraction of the input power that is reflected (give a numerical answer).

b) Find the total voltage V_T at the point $z=0$ in the figure above in terms of the input wave amplitude V_{+1}.

c) The total voltage V_T found in part (b) must also equal V_{+2} and V_{+3} since they are in parallel. Knowing this, find the fraction of the input power transmitted in the each of the two output transmission lines (give numerical answers). Do all your fractions (reflected and transmitted) add up to unity? They should.

d) Suppose you could choose the impedances Z_{02} and Z_{03} of the output transmission lines to be whatever you wanted. Choose these values such that you simultaneously satisfy the following two conditions:

i) No fraction of the input power is reflected
ii) The output transmission line with impedance Z_{02} has twice as much power going into it as the transmission line with impedance Z_{03}.

Problem 8.6: (Plasma cut-off frequency)

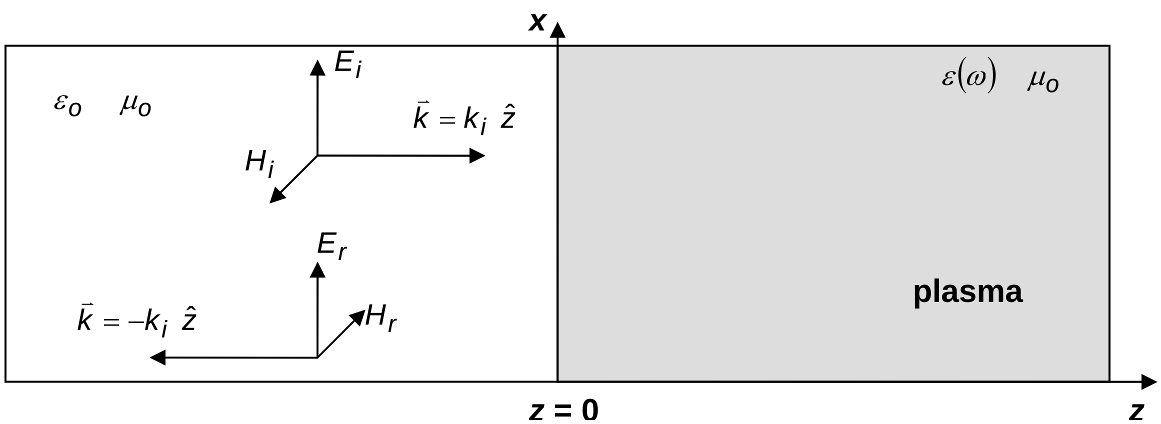
In the lectures you were told that if the frequency ω of an electromagnetic wave is less than the plasma frequency ω_p then the wave is completely reflected at the surface of a plasma. In this problem, you will explore this further and see if the above statement holds in all cases. Consider an electromagnetic wave given by:

$$\hat{x}\, E_i\, e^{-jk_i z}$$

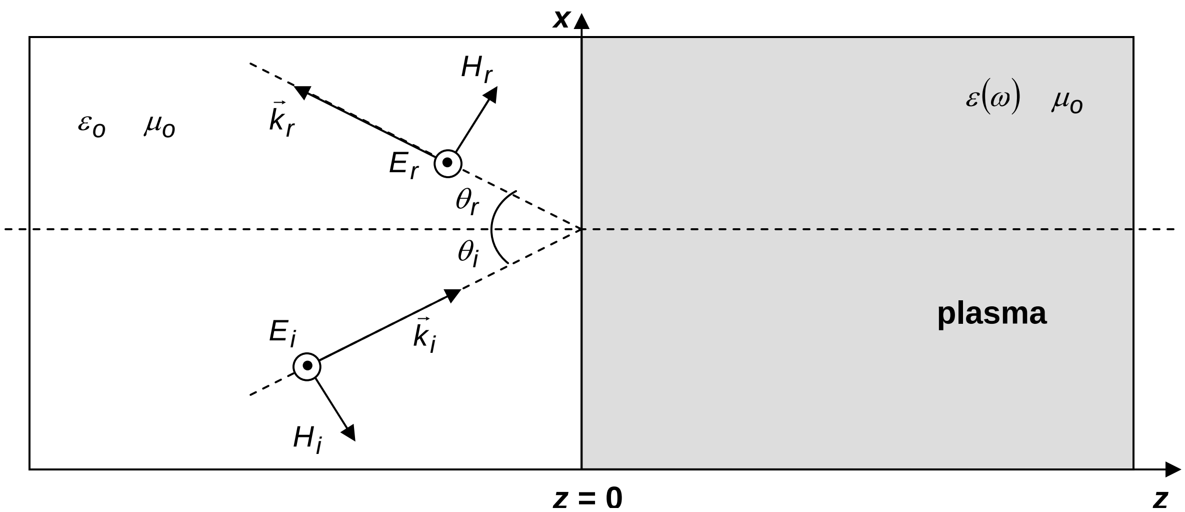
incident **normally** from free-space on a plasma whose permittivity is given by the relation:

$$\varepsilon(\omega) = \varepsilon_o \left(1 - \frac{\omega_p^2}{\omega^2}\right)$$

In the previous homework you showed that when the frequency ω of the electromagnetic wave is less than the plasma frequency ω_p, the magnitude of the reflection coefficient $\Gamma = \dfrac{E_r}{E_i}$ is unity.



a) The frequency below which a wave is completely reflected (i.e. $|\Gamma| = 1$) at the surface of a plasma is called the cut-off frequency ω_c. In the previous homework you essentially showed that the cut-ff frequency ω_c is just the plasma frequency ω_p **provided the wave is incident normally on the plasma**. Now suppose a TE-wave is incident at an angle of incidence θ_i as shown in the figure below.



Find the cut-off frequency ω_c as a function of the angle of incidence θ_i for the TE-Wave. Is the cut-off frequency ω_c now larger or smaller than the plasma frequency ω_p ? **Hint:** think what is required to get $|\Gamma| = 1$. Although this problem is not about total internal reflection, you may want to study carefully how $|\Gamma|$ becomes unity in the case of total internal reflection.

b) Same as part (a) but now suppose a TM-wave is incident at an angle of incidence θ_i . Find the cut-off frequency ω_c as a function of the angle of incidence θ_i for the TM-wave. Is the cut-off frequency ω_c now larger or smaller than the plasma frequency ω_p ?