**ECE 303: Electromagnetic Fields and Waves**

**Fall 2007**

**Homework 7** **Due on Oct. 12, 2007 by 5:00 PM**

## Reading Assignments:

i) Review the lecture notes.
ii) Review sections 4.1-4.3, 5.1-5.2, 5.4, 6.1, 6.3-6.4, paperback book *Electromagnetic Waves*. These sections also include the material to be covered in the next two weeks of the class.

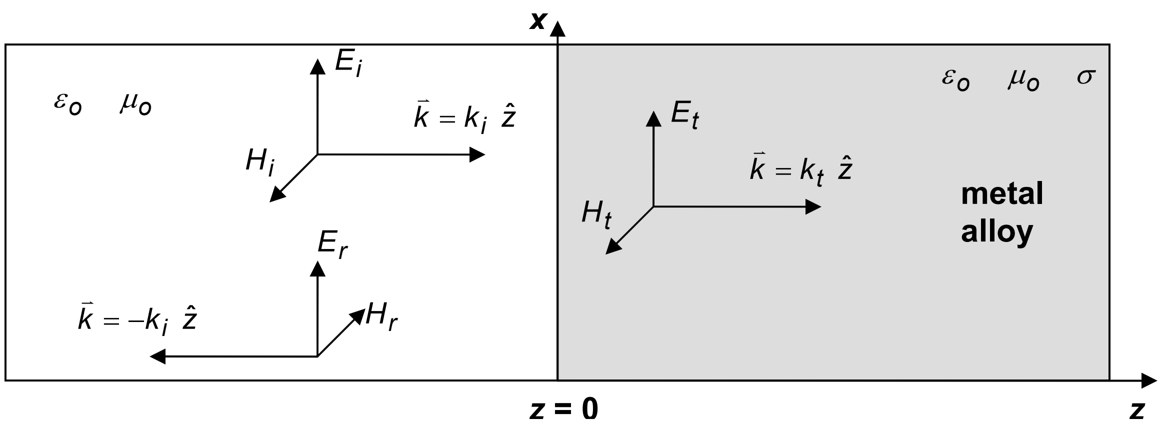
**Special Note:** Graders have been instructed to take off points (as much as 50%) if proper units are not included in your answers. You must specify the correct units with your numerical answers.

## Problem 7.1: (Reflection and power dissipation for a conductive medium)

Consider an electromagnetic wave given by:

$$\vec{E}_i(\vec{r}) = \hat{x}\, E_o\, e^{-jk_i z}$$

with a frequency equal to 1 GHz and incident from free space upon a metal alloy. The conductivity $\sigma$ of the metal alloy is $\sigma = 10$ S/m and the dielectric permittivity of the alloy can be taken as $\varepsilon_o$.




a) Find the magnitude of the reflection coefficient $\Gamma$.

b) Find the time-average **power dissipated per unit area** in the metal alloy. This can be found by subtracting the Poynting vector of the reflected wave from that of the incident wave. Assume that the incident wave carries a power per unit area equal to 1 Watt/m$^2$. You need to give a numerical value with proper units (not just an expression) as an answer. What fraction of the incident power per unit area is dissipated in the metal alloy?

c) The power dissipated inside the metal alloy can also be found by a more **direct** calculation. You can calculate the time-average power dissipated per unit volume inside the metal alloy using:

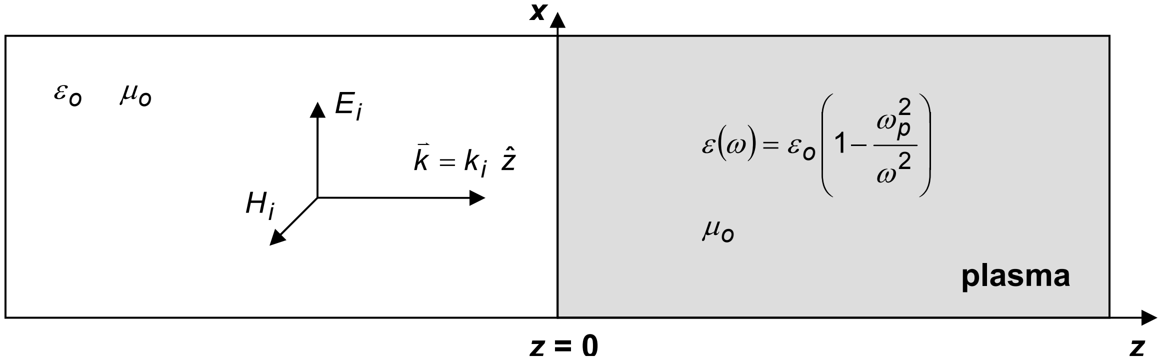
$$\frac{1}{2}\text{Re}\left[\vec{J}(\vec{r}).\vec{E}_t^*(\vec{r})\right]$$

and then integrate over $z$ from 0 to $+\infty$ to get the time-average power dissipated per unit area, i.e.:

$$\frac{1}{2}\int_0^\infty \mathrm{Re}\left[\vec{J}(\vec{r}).\vec{E}_t^*(\vec{r})\right]dz$$

To calculate the above integral you will first have to have to find the transmission coefficient and the complex wavevector inside the metal alloy. Evaluate the integral and give a numerical answer and show that it is the same as that calculated in part (b) above.

### Problem 7.2: (Reflection off a plasma for $\omega < \omega_p$)




Consider an electromagnetic wave given by:

$$\vec{E}_i(\vec{r}) = \hat{x}\ E_o\ e^{-jk_i z}$$

incident from free space on a plasma, as shown above.

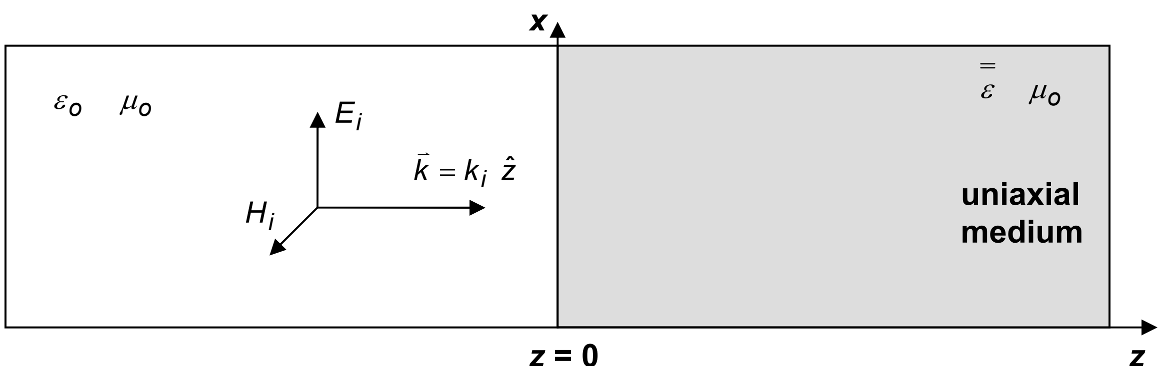
a) Show that the magnitude of the reflection coefficient $\Gamma$ is unity when the frequency $\omega$ of the incident wave is less than the plasma frequency $\omega_p$. If the magnitude of the reflection coefficient $\Gamma$ is unity then this means that all the incident power is reflected.

### Problem 7.3: (Reflections off the surface of a uniaxial medium)

Consider an electromagnetic wave incident from free space upon a uniaxial medium. The medium is specified by the permittivity tensor:

$$\overline{\overline{\varepsilon}} = \varepsilon_o\begin{bmatrix} 9 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

In this problem you will look at reflections from the surface of the uniaxial medium – something that we ignored previously.



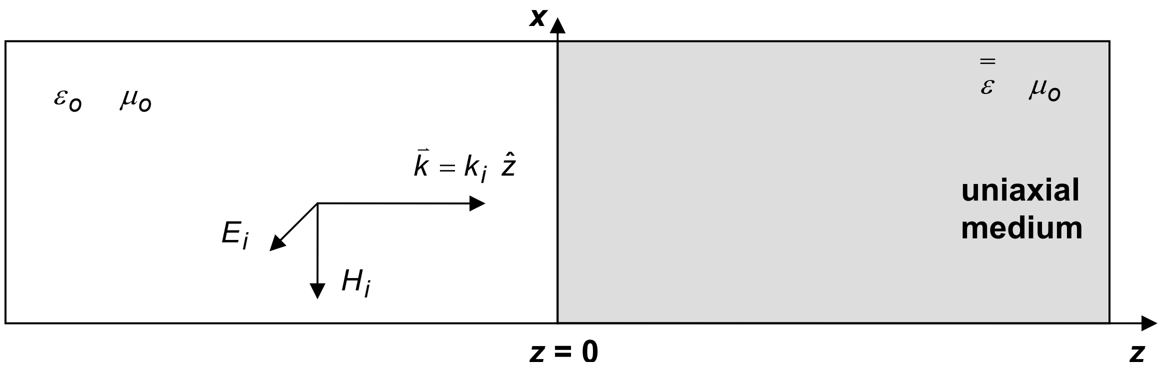
a) Assume first that the incident wave is x-polarized, as shown in the figure above:

$\vec{E}_i(\vec{r}) = \hat{x}\ E_o\ e^{-jk_i z}$

One can write the reflected wave as:

$\vec{E}_r(\vec{r}) = \hat{x}\ \Gamma_x\ E_o\ e^{+jk_i z}$

What is the reflection coefficient $\Gamma_x$? Give a numerical value.



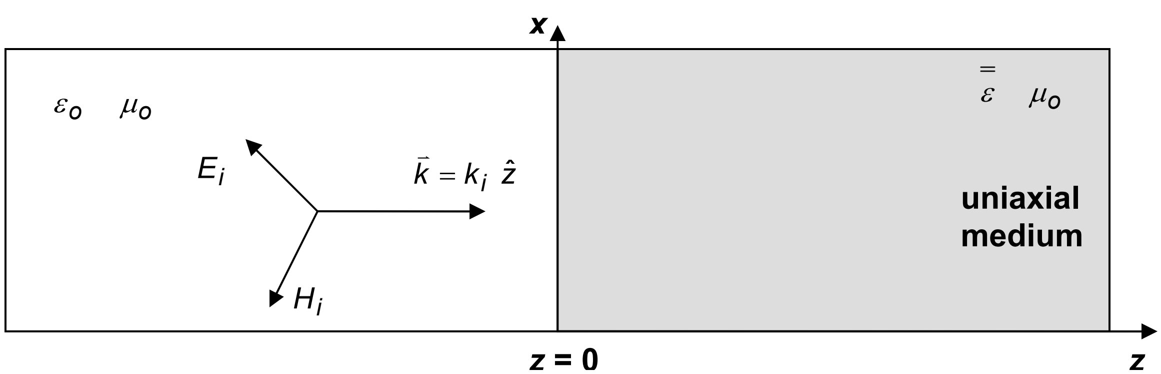

b) Assume now that the incident wave is y-polarized, as shown in the figure above:

$\vec{E}_i(\vec{r}) = \hat{y}\ E_o\ e^{-jk_i z}$

One can write the reflected wave as:

$\vec{E}_r(\vec{r}) = \hat{y}\ \Gamma_y\ E_o\ e^{+jk_i z}$

What is the reflection coefficient $\Gamma_y$? Give a numerical value.




Now assume that the incident wave has both x- and y-components, as shown in the figure above:

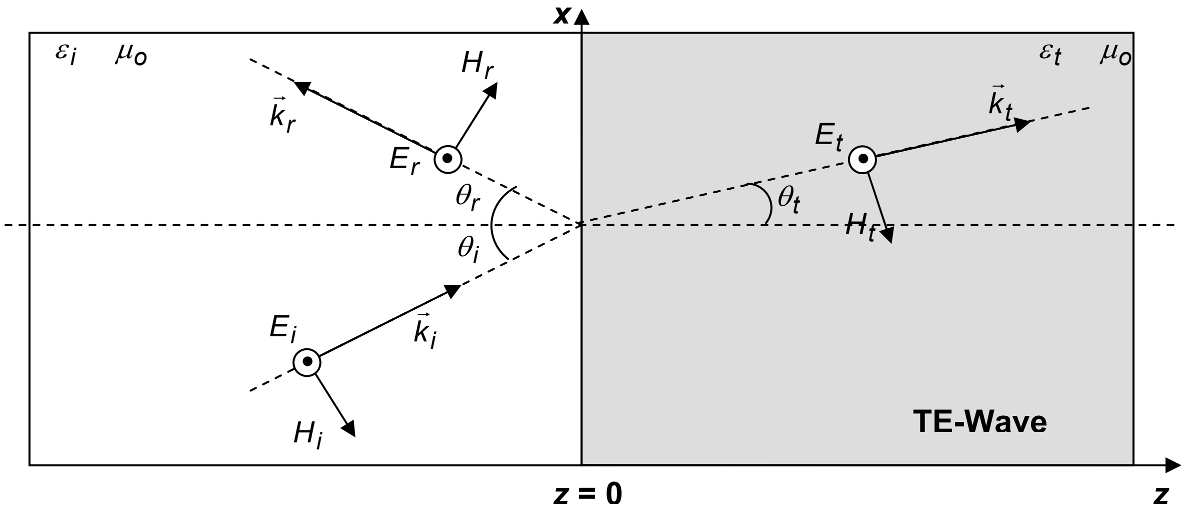
$$\vec{E}_i(\vec{r}) = \left(\frac{\hat{x} + \hat{y}}{2}\right) E_o\ e^{-jk_i z}$$

c) Write an expression for the E-field of the reflected wave:

$\vec{E}_r(\vec{r}) = ?$

d) What is the angle between the E-field polarization directions of the incident and reflected waves? Give a numerical value. Has the E-field rotated upon reflection?

## Problem 7.4: (TE and TM reflections and transmissions)




Consider an incident TE-wave for which the E-field and the H-field are:

$\hat{y}\, E_{iy}\, e^{-j\vec{k}_i \cdot \vec{r}}$ and $\left(\hat{x}\, H_{ix} + \hat{z}\, H_{iz}\right) e^{-j\vec{k}_i \cdot \vec{r}}$

The E-fields of the reflected and transmitted waves are:

$\hat{y}\, E_{ry}\, e^{-j\vec{k}_r \cdot \vec{r}}$ and $\hat{y}\, E_{ty}\, e^{-j\vec{k}_t \cdot \vec{r}}$

The H-fields of the reflected and transmitted waves are:

$\left(\hat{x}\, H_{rx} + \hat{z}\, H_{rz}\right) e^{-j\vec{k}_r \cdot \vec{r}}$ and $\left(\hat{x}\, H_{tx} + \hat{z}\, H_{tz}\right) e^{-j\vec{k}_t \cdot \vec{r}}$ For the reflected and transmitted E-fields the reflection and transmission coefficients were calculated in the lecture notes, and are as follows:

$$\frac{E_{ry}}{E_{iy}} = \frac{\eta_t \cos(\theta_i)/\eta_i \cos(\theta_t) - 1}{\eta_t \cos(\theta_i)/\eta_i \cos(\theta_t) + 1} \qquad \frac{E_{ty}}{E_{iy}} = \frac{2\,\eta_t \cos(\theta_i)/\eta_i \cos(\theta_t)}{\eta_t \cos(\theta_i)/\eta_i \cos(\theta_t) + 1} \qquad \text{———} \qquad (1)$$

a) Find the following ratios (which are the reflection and transmission coefficients for each component of the H-field):

i) $\dfrac{H_{rx}}{H_{ix}} = ?$ ii) $\dfrac{H_{tz}}{H_{iz}} = ?$

**Hint:** You can express the incident, transmitted, and reflected H-field components in terms of the incident, transmitted, and reflected E-field y-components, respectively, and then use the results given above in (1).

Now consider an incident TM-wave for which the H-field and the E-field are:

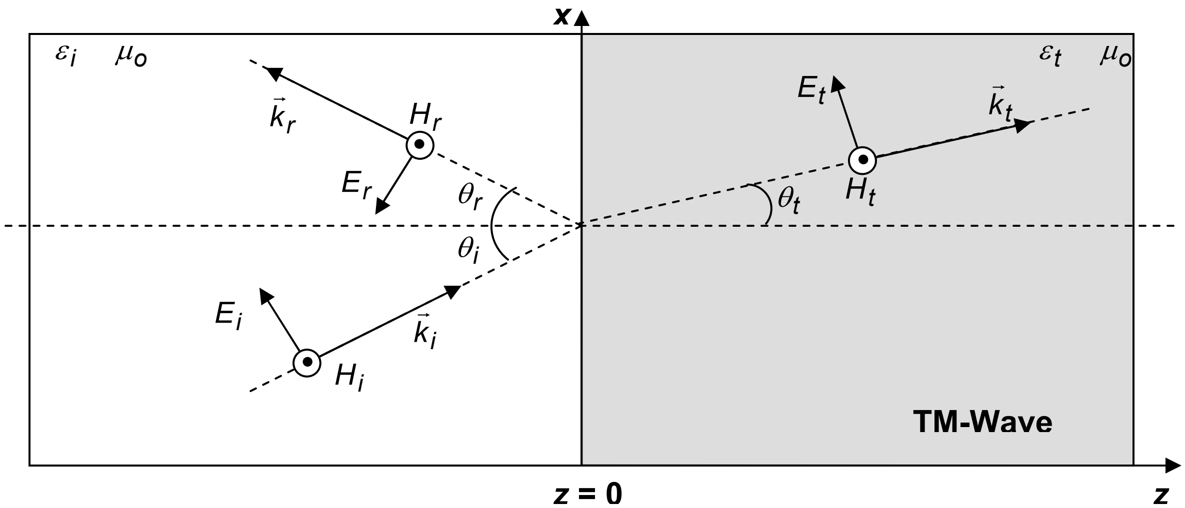
$\hat{y}\, H_{iy}\, e^{-j\vec{k}_i \cdot \vec{r}}$ and $\left(\hat{x}\, E_{ix} + \hat{z}\, E_{iz}\right) e^{-j\vec{k}_i \cdot \vec{r}}$

The H-fields of the reflected and transmitted waves are:

$\hat{y}\, H_{ry}\, e^{-j\vec{k}_r \cdot \vec{r}}$ and $\hat{y}\, H_{ty}\, e^{-j\vec{k}_t \cdot \vec{r}}$

The E-fields of the reflected and transmitted waves are:

$\left(\hat{x}\, E_{rx} + \hat{z}\, E_{rz}\right) e^{-j\vec{k}_r \cdot \vec{r}}$ and $\left(\hat{x}\, E_{tx} + \hat{z}\, E_{tz}\right) e^{-j\vec{k}_t \cdot \vec{r}}$




For the reflected and transmitted H-fields the reflection and transmitted coefficients were calculated in the lecture notes, and are as follows:

$$\frac{H_{ry}}{H_{iy}} = \frac{\eta_i \cos(\theta_i)/\eta_t \cos(\theta_t) - 1}{\eta_i \cos(\theta_i)/\eta_t \cos(\theta_t) + 1} \qquad \frac{H_{ty}}{H_{iy}} = \frac{2\,\eta_i \cos(\theta_i)/\eta_t \cos(\theta_t)}{\eta_i \cos(\theta_i)/\eta_t \cos(\theta_t) + 1} \qquad \text{———} \qquad (2)$$

b) Find the following ratios (which are the reflection and transmission coefficients for each component of the E-field):

i) $\dfrac{E_{rz}}{E_{iz}} = ?$ ii) $\dfrac{E_{tx}}{E_{ix}} = ?$

**Hint:** You can express the incident, transmitted, and reflected E-field components in terms of the incident, transmitted, and reflected H-field y-components, respectively, and then use the results given above in (2).