ECE 303 Homework #5 Solutions

(Farhan Rana)

5.1

a) $\vec{E}(\vec{r}) = \hat{n}\, E_o\, e^{-j\vec{k}\cdot\vec{r}}$ $\quad \left\{ \nabla = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z} \right\}$

$$\nabla\times\vec{E}(\vec{r}) = \left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right)\times \hat{n}\, e^{-j(k_x x + k_y y + k_z z)}$$

$$= e^{-j(k_x x + k_y y + k_z z)}\, (-j)\left[\hat{x}\, k_x + \hat{y}\, k_y + \hat{z}\, k_z\right]\times\hat{n}$$

$$= -j\,\vec{k}\times\hat{n}\; E_o e^{-j\vec{k}\cdot\vec{r}}$$

b) $$\nabla^2\vec{E}(\vec{r}) = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\hat{n}\, E_o e^{-j(k_x x + k_y y + k_z z)}$$

$$= -(k_x^2 + k_y^2 + k_z^2)\,\hat{n}\, E_o e^{-j\vec{k}\cdot\vec{r}} = -k^2\,\hat{n}\, E_o e^{-j\vec{k}\cdot\vec{r}}$$

c) $$\nabla\cdot\vec{E}(\vec{r}) = \left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right)\cdot\hat{n}\, e^{-j(k_x x + k_y y + k_z z)}$$

$$= e^{-j(k_x x + k_y y + k_z z)}\,(-j)\left(\hat{x}\, k_x + \hat{y}\, k_y + \hat{z}\, k_z\right)\cdot\hat{n} = -j\,\vec{k}\cdot\hat{n}\; E_o e^{-j\vec{k}\cdot\vec{r}}$$

5.2

i) $$\vec{E}(\vec{r}) = \left(\hat{z} + e^{j\frac{\pi}{2}}\hat{y}\right)e^{-jkx} = e^{j\frac{\pi}{2}}\left(\hat{y} + e^{-j\frac{\pi}{2}}\hat{z}\right)e^{-jkx}$$

$\Rightarrow$ right-hand circularly polarized

ii) $$\vec{E}(\vec{r}) = (2+j)\left[\hat{x} + \frac{(3j+1)}{(2+j)}\hat{z}\right]E_o e^{jky} \qquad \frac{3j+1}{2+j} = 1+j = \sqrt{2}\, e^{j\frac{\pi}{4}}$$

$\Rightarrow$ left-hand elliptically polarized

iii) $$\vec{H}(\vec{r}) = (\hat{x} - j\hat{y})\, H_o e^{jkz} \Rightarrow \vec{E}(\vec{r}) = \frac{\nabla\times\vec{H}(\vec{r})}{j\omega\varepsilon_o} = -\eta_o(-\hat{z})\times\vec{H}(\vec{r})$$

$$\vec{E}(\vec{r}) = (\hat{y} + j\hat{x})\,\eta_o H_o\, e^{jkz}$$

$\Rightarrow$ left-hand circularly polarized

iv) $\vec{E}(\vec{r}) = (-\hat{x}+\hat{y}) E_o e^{-jk\left(\frac{x+y}{\sqrt{2}}\right)} \Rightarrow \vec{k} = k\,\frac{(\hat{x}+\hat{y})}{\sqrt{2}}$

linearly - polarized.

b)

i) $\vec{E}(\vec{r}) = (j\hat{y}+\hat{z}) E_o e^{-jkx}$ $\quad \left\{ \vec{k} = k\hat{x} \right.$

$$\vec{H}(\vec{r}) = \frac{\hat{k}\times\vec{E}(\vec{r})}{\eta_o} = \frac{\hat{x}\times\vec{E}(\vec{r})}{\eta_o} = (j\hat{z}-\hat{y})\frac{E_o}{\eta_o} e^{-jkx}$$

ii) $\vec{E}(\vec{r}) = \left[\hat{x}(2+j) + (3j+1)\hat{z}\right] E_o e^{jky}$ $\quad \left\{ \vec{k} = k\hat{y} \right.$

$$\vec{H}(\vec{r}) = \left[\hat{z}(2+j) - (3j+1)\hat{x}\right]\frac{E_o}{\eta_o} e^{jky}$$

iii) $\vec{H}(\vec{r}) = \left[\hat{x} - j\hat{y}\right] H_o e^{jkz}$

$$\vec{E}(\vec{r}) = (\hat{y}+j\hat{x})\eta_o H_o e^{jkz}$$

iv) $\vec{E}(\vec{r}) = (-\hat{x}+\hat{y}) E_o e^{-jk\left(\frac{x+y}{\sqrt{2}}\right)}$ $\quad \left\{ \vec{k} = k\left(\frac{\hat{x}+\hat{y}}{\sqrt{2}}\right) \right.$

$$\vec{H}(\vec{r}) = \frac{\nabla\times\vec{E}(\vec{r})}{-j\omega\mu_o} = \frac{\hat{k}\times\vec{E}(\vec{r})}{\eta_o} = \hat{z}\sqrt{2}\,\frac{E_o}{\eta_o} e^{-jk\left(\frac{x+y}{\sqrt{2}}\right)}$$

c)

i) $\langle\vec{S}(\vec{r},t)\rangle = \frac{1}{2}\mathrm{Re}\left\{\vec{E}(\vec{r})\times\vec{H}^*(\vec{r})\right\} = \frac{1}{2}\mathrm{Re}\left\{\vec{S}(\vec{r})\right\}.$

$$\vec{S}(\vec{r}) = \vec{E}(\vec{r})\times\vec{H}^*(\vec{r}) = \hat{x}\,2\frac{|E_o|^2}{\eta_o} \Rightarrow \langle\vec{S}(\vec{r},t)\rangle = \hat{x}\frac{|E_o|^2}{\eta_o}$$

ii) $\vec{S}(\vec{r}) = \vec{E}(\vec{r})\times\vec{H}^*(\vec{r}) = \left[-\hat{y}(5) - \hat{y}(10)\right]\frac{|E_o|^2}{\eta_o} = -15\frac{|E_o|^2}{\eta_o}\hat{y}$

$$\Rightarrow \langle\vec{S}(\vec{r},t)\rangle = \frac{1}{2}\mathrm{Re}\left\{\vec{S}(\vec{r})\right\} = -\frac{15}{2}\frac{|E_o|^2}{\eta_o}\hat{y}$$

iii) $\vec{S}(\vec{r}) = \vec{E}(\vec{r}) \times \vec{H}^*(\vec{r}) = -\hat{z}\, 2\, \eta_0 |H_0|^2$

$\langle \vec{S}(\vec{r},t) \rangle = \frac{1}{2} Re\{ \vec{S}(\vec{r}) \} = -\eta_0 |H_0|^2 \hat{z}$

iv) $\vec{S}(\vec{r}) = \vec{E}(\vec{r}) \times \vec{H}^*(\vec{r}) = \left(\frac{\hat{y} + \hat{x}}{\sqrt{2}} \right) \frac{2|E_0|^2}{\eta_0}$

$\langle \vec{S}(\vec{r},t) \rangle = \frac{1}{2} Re\{ \vec{S}(\vec{r}) \} = \frac{(\hat{y} + \hat{x})}{\sqrt{2}} \frac{|E_0|^2}{\eta_0}$