School of Electrical and Computer Engineering, Cornell University

ECE 303: Electromagnetic Fields and Waves

Fall 2007

Homework 5 **Due on Sep. 28, 2007 by 5:00 PM**

Reading Assignments:

i) Review the lecture notes.
ii) Review sections 1.1-1.6 of the paperback book *Electromagnetic Waves*. We have officially started the paperback textbook and will follow it till the end of the semester.

Problem 5.1: (Vector Calculus and Plane Waves)

Consider a plane wave whose E-field vector phasor is given by the following expression:

$$\vec{E}(\vec{r}) = \hat{n}\, E_o\, e^{-j\,\vec{k}\,.\,\vec{r}}$$

Where $\vec{k}$ and the unit vector $\hat{n}$ can be decomposed into components as follows:

$$\vec{k} = k_x\,\hat{x} + k_y\,\hat{y} + k_z\,\hat{z}$$
$$\hat{n} = n_x\,\hat{x} + n_y\,\hat{y} + n_z\,\hat{z}$$

Prove the following three relations:

a) $\nabla \times \vec{E}(\vec{r}) = \left(-j\,\vec{k} \times \hat{n}\right) E_o\, e^{-j\vec{k}\,.\,\vec{r}}$
b) $\nabla^2\,\vec{E}(\vec{r}) = \left(-k^2\right)\hat{n}\,E_o\, e^{-j\,\vec{k}\,.\,\vec{r}}$
c) $\nabla\,.\vec{E}(\vec{r}) = \left(-j\,\vec{k}\,.\hat{n}\right) E_o\, e^{-j\vec{k}\,.\,\vec{r}}$

Problem 5.2: (Polarization of Plane Waves)

a) Find the Polarization (i.e., linear, circular, or elliptical, and left-handed or right-handed) of the following plane waves:

i) $\vec{E}(\vec{r}) = \left(j\,\hat{y} + \hat{z}\right) E_o\, e^{-jk\,x}$
ii) $\vec{E}(\vec{r}) = \left[\hat{x}\,(2 + j) + \hat{z}\,(3\,j + 1)\right] E_o\, e^{jk\,y}$
iii) $\vec{H}(\vec{r}) = \left[\hat{x} - j\,\hat{y}\right] H_o\, e^{jk\,z}$
iv) $\vec{E}(\vec{r}) = \left(-\hat{x} + \hat{y}\right) E_o\, e^{-jk\,\frac{(x+y)}{\sqrt{2}}}$

b) Find the magnetic field (or the electric field - whichever is not given) for the following plane waves:

i) $\vec{E}(\vec{r}) = (j\,\hat{y} + \hat{z})\,E_o\,e^{-jk\,x}$

ii) $\vec{E}(\vec{r}) = [\hat{x}\,(2 + j) + \hat{z}\,(3\,j + 1)]\,E_o\,e^{jk\,y}$

iii) $\vec{H}(\vec{r}) = [\hat{x} - j\,\hat{y}]\,H_o\,e^{jk\,z}$

iv) $\vec{E}(\vec{r}) = (-\hat{x} + \hat{y})\,E_o\,e^{-jk\frac{(x+y)}{\sqrt{2}}}$

c) Find the time-average power flow per unit area (magnitude and direction) carried by the following plane waves:

i) $\vec{E}(\vec{r}) = (j\,\hat{y} + \hat{z})\,E_o\,e^{-jk\,x}$

ii) $\vec{E}(\vec{r}) = [\hat{x}\,(2 + j) + \hat{z}\,(3\,j + 1)]\,E_o\,e^{jk\,y}$

iii) $\vec{H}(\vec{r}) = [\hat{x} - j\,\hat{y}]\,H_o\,e^{jk\,z}$

iv) $\vec{E}(\vec{r}) = (-\hat{x} + \hat{y})\,E_o\,e^{-jk\frac{(x+y)}{\sqrt{2}}}$