ECE 303: Electromagnetic Fields and Waves

Fall 2007

Homework 2   **Due on Sep. 07, 2007 by 5:00 PM**

Reading Assignments:

i) Review the lecture notes.

ii) Relevant sections of the online *Haus and Melcher* book for this week are 4.0-4.6. Note that the book contains more material than you are responsible for in this course. Determine relevance by what is covered in the lectures and the recitations. The book is meant for those of you who are looking for more depth and details.

Spherically or Cylindrically Symmetric Solutions of Laplace's Equation

Spherical Coordinate System	Cylindrical Coordinate System
$\phi(r) = \dfrac{A}{r} + B$	$\phi(r) = A\ln(r) + B$

Problem 2.1: (Electric charge above a perfect metal strip)

Consider a point charge sitting on the z-axis at location $z = +d/2$ above an infinite perfect metal strip (infinite in the *x-y* plane and of thickness $d/4$), as shown in the figure below.
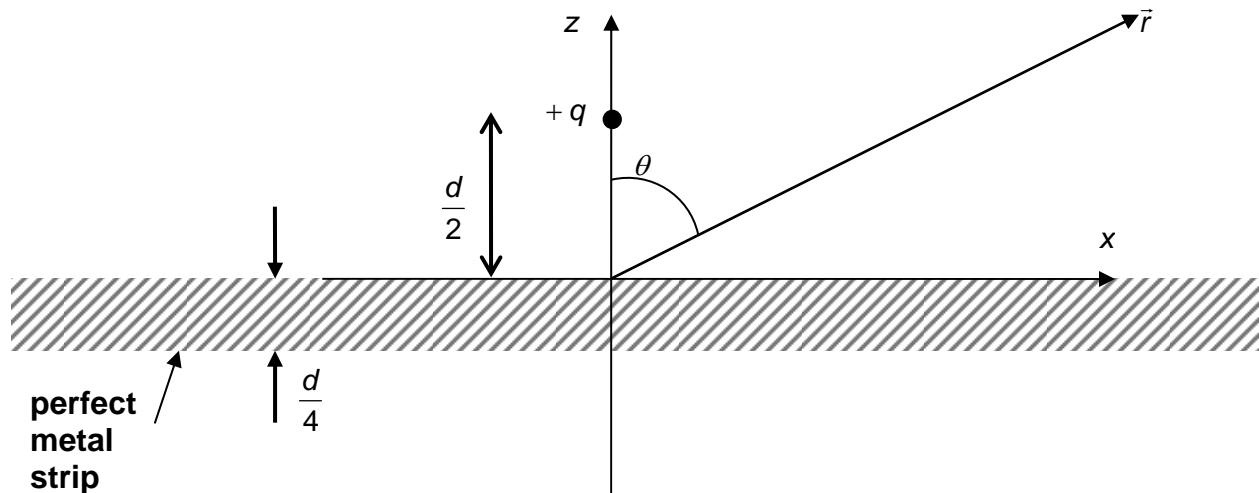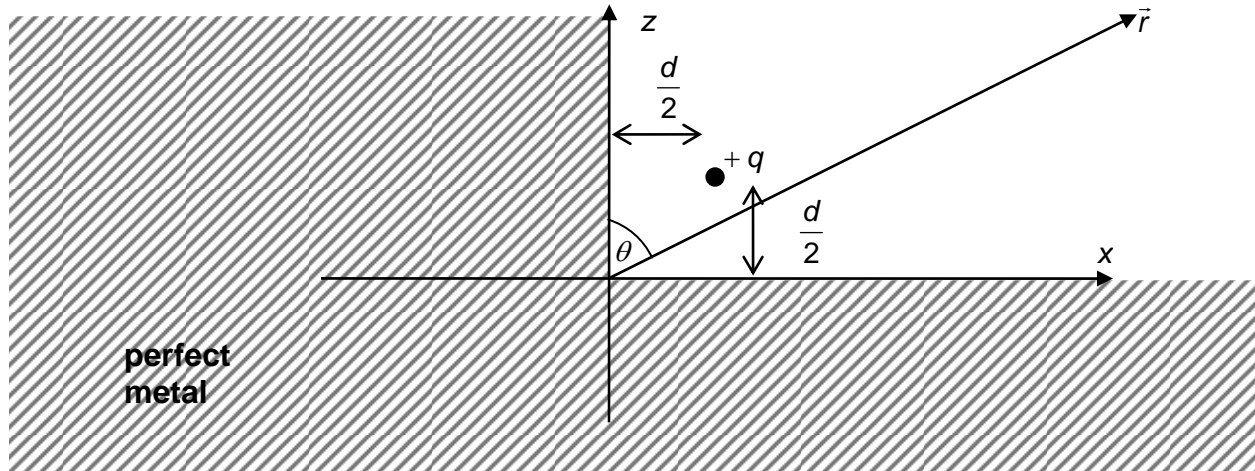

a) Find the potential $\phi(\vec{r})$ outside the perfect metal at a location $\vec{r}$ far away from the charge (where $|\vec{r}| \gg d$). Use spherical coordinates. You need to find the potential for both $z < 0$ and $z > 0$.

b) Find the electric field $\vec{E}(\vec{r})$ from your result in part (a).

c) Find the position dependent surface charge density (units: Coulombs/m²) on the upper and lower surfaces of the metal strip.

d) From your answer in part (c) find the total charge (units: Coulombs) on the upper surface of the metal strip.

Problem 2.2: (Electric charge near perfect metal planes)

Consider a point charge placed near the corner of perfect metal planes, as show in the figure below. The metal planes are infinite in the y-direction. The point charge is placed at $y = 0$.


a) Using the method of images draw the image charges corresponding to the point charge and indicate the location, orientation and magnitude of these image charges in a sketch.

b) Find the potential $\phi(\vec{r})$ outside the perfect metal at a location $\vec{r}$ far away from the charge (where $|\vec{r}| \gg d$) in the x-z plane (i.e. for the angle ϕ in the spherical co-ordinates equal to zero). Use spherical coordinates.

Hints:
1) Do not attempt a full 3D solution – just the solution in the 2D x-z plane.
2) Assume that $|\vec{r}| \gg d$.
3) At some stage in this problem you will have to do a Taylor expansion up to the second order since the first order terms will all cancel.
4) In your final answer, the potential should have a $1/r^3$ distance dependence and a $\sin(2\theta)$ angular dependence.
5) The answer you will obtain corresponds to what is called a charge "quadrupole".

Problem 2.3: (A point charge surrounded by a metal spherical shell)

Consider a point charge of $+q$ Coulombs surrounded by a perfect metal spherical shell, as shown below.
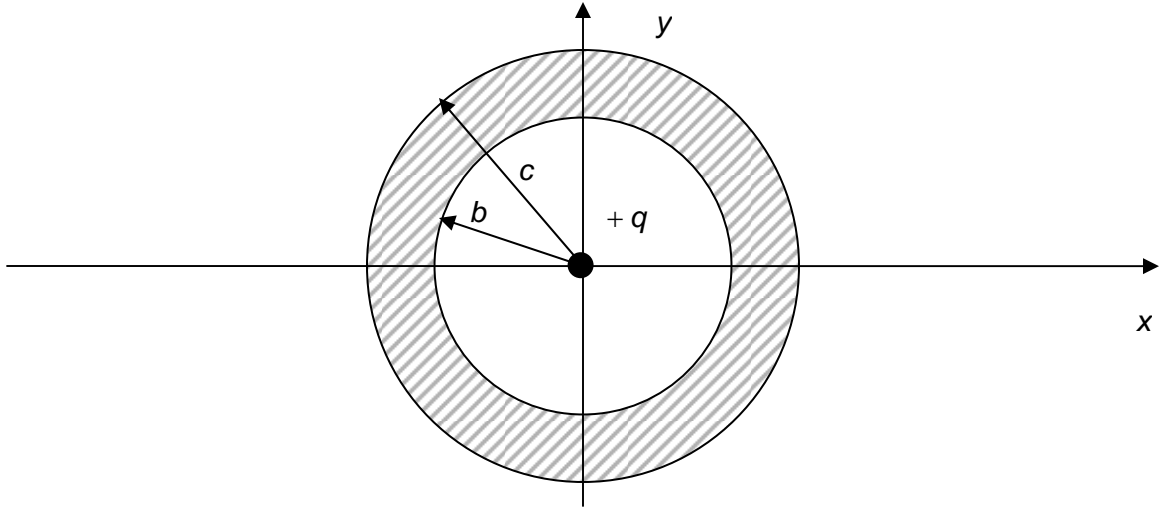
a) Find the electric field vector $E(\vec{r})$ in the three regions: (i) $0 < r < b$ (ii) $b < r < c$ (iii) $c < r < \infty$.

b) Find the **surface charge densities** (sign and magnitude) on both the inner and outer surfaces of the metal spherical shell and then find the **total charge** (sign and magnitude) on both the inner and outer surfaces of the metal spherical shell.

Now suppose the spherical shell is grounded (i.e. its potential is fixed to be zero w.r.t. infinity) as shown below. The problem is now going to be tricky so be very careful.
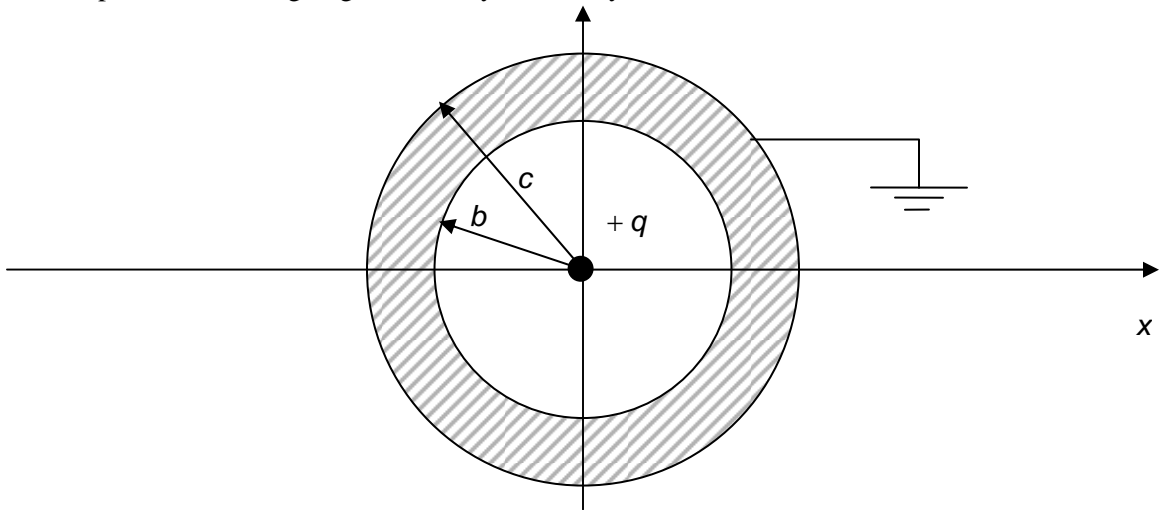
c) Find the potential $\phi(\vec{r})$ in the three regions: (i) $0 < r \le b$ (ii) $b \le r \le c$ (iii) $c \le r \le \infty$.

d) Find the electric field vector $E(\vec{r})$ in the three regions: (i) $0 < r < b$ (ii) $b < r < c$ (iii) $c < r < \infty$. Hint: Use your results from part (c).

e) Find the **surface charge densities** (sign and magnitude) on both the inner and outer surfaces of the metal spherical shell and then find the **total charge** (sign and magnitude) on both the inner and outer surfaces of the metal spherical shell. Hint: Use your results from part (d).

f) Compare your results from parts (b) and (e) and explain what happened when the shell was grounded.

Problem 2.4: (A metal sphere surrounded by a metal spherical shell)

Consider a perfect metal sphere surrounded by a perfect metal spherical shell and connected to a voltage source as shown below. Completely ignore the physical presence of the voltage source and the connecting wires other than the fact that they establish a fixed potential difference between the inner sphere and the outer spherical shell.
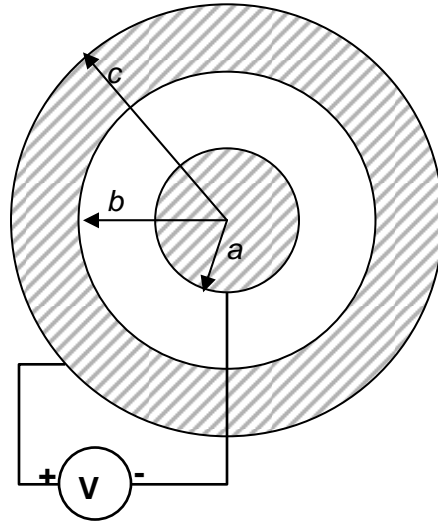
a) Find the potential $\phi(\vec{r})$ in the region $a \leq r \leq b$. Make sure your solution satisfies all relevant boundary conditions.

b) Sketch the potential $\phi(\vec{r})$ for $0 \leq r \leq c$ and label your sketch.

c) Find the electric field vector $E(\vec{r})$ in the three regions: (i) $0 < r < a$ (ii) $a < r < b$ (iii) $b < r < c$.

d) Find the surface charge densities (sign and magnitude) on the inner surface of the metal spherical shell and on the outer surface of the inner metal sphere.

e) Find the capacitance C (units: Farads) between the inner metal sphere and the outer metal shell.

Problem 2.5: (Parallel plates with volume charge density)

Consider two perfect metal parallel plates connected to a voltage source as shown in the figure below. The distance between the plates is much smaller than the size of the plates and so the problem is essentially one dimensional. The space between the plates if filled with a uniform volume charge density given by ρ.

a) Find the potential $\phi(x)$ in between the plates. Your answer must satisfy all the relevant boundary conditions.

b) Find the electric field $E_x(x)$ in between the plates.

c) Find the surface charge densities (sign and magnitudes) on the inner surfaces of the left and the right metal plates.

d) Find the capacitance C (units: Farads) between the left and the right plates. Note that the correct formula for the capacitance is dQ/dV rather than Q/V.

e) When the applied potential V is 0 Volts, what is the total combined charge on the inner surface of both the plates and how does it compare to the total charge in the region between the plates?
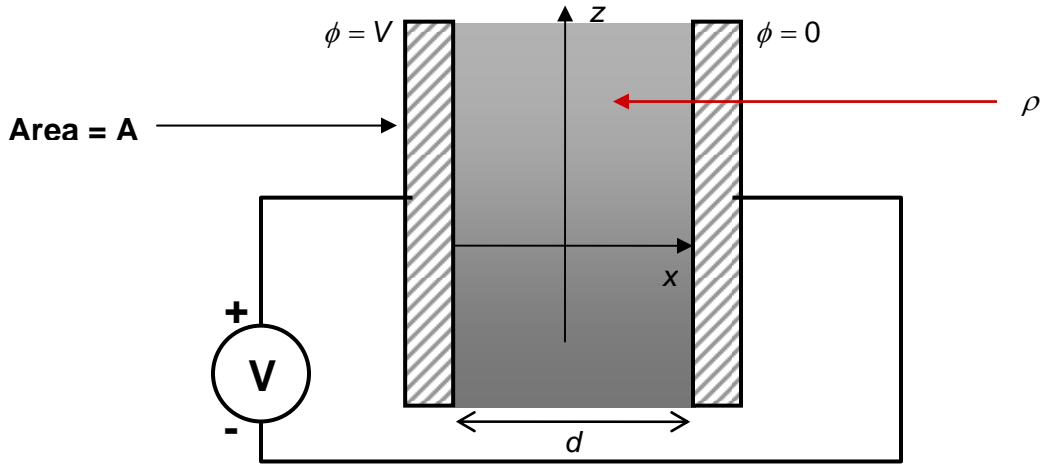
Problem 2.6: (Capacitance between a thin metal wire and a metal ground plane)

Consider a thin metal wire of radius a over an infinite perfect metal ground plane, as shown in the figure below. The distance of the wire from the ground plane is much larger than the radius of the wire (i.e. $d \gg a$). The structure is infinite in the z-direction. A voltage source establishes a fixed potential difference between wire and the metal ground plane. Completely ignore the physical presence of the voltage source and the connecting wires other than the fact that they establish a fixed potential difference between the metal wire and the ground plane. In this problem you will need to find the **capacitance per unit length** (i.e. the length in the z-direction) between the wire and the ground plane.
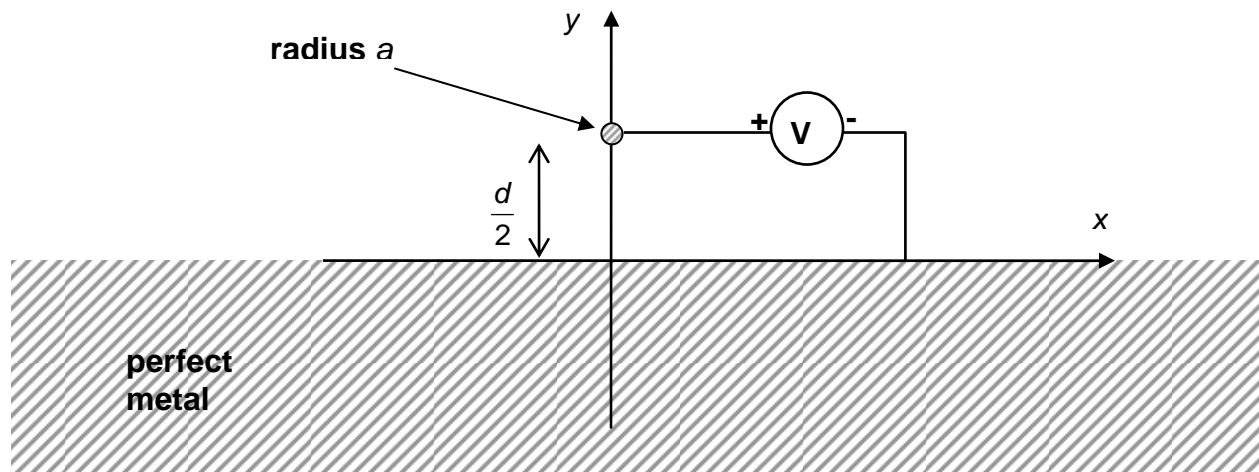
a) Start by assuming that there is a charge density $+\lambda$ on the metal wire per unit length. Sketch the image wire using the method of images. Indicate the orientation and the charge per unit length that the image wire carries on a sketch.

b) Using results from the lecture notes, write an expression that gives the potential $\phi(\bar{r})$ everywhere in the region outside the perfect metal (assuming $d \gg a$).

c) Write an expression that relates the potential difference V between the wire and the ground plane to the charge density per unit length $+\lambda$ on the wire.

d) Find the capacitance per unit length C (units: Farads/m) between the wire and the ground plane by taking the ratio of the charge per unit length on the wire to the applied potential difference V between the wire and the metal plane.