Lecture 31

Wire Antennas

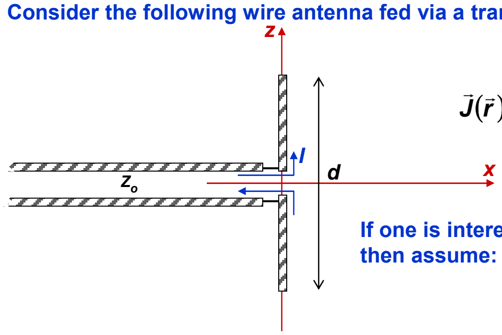
In this lecture you will learn:

- Generation of radiation by real wire antennas

  Short dipole antennas
  Half-wave dipole antennas
  Three-half-wave dipole antennas
  Small wire loop antennas – magnetic dipole antennas

Center Fed Wire Antenna

Consider the following wire antenna fed via a transmission line:



$$\vec{J}(\vec{r}) = \hat{z}\, I(z)\, \delta(x)\, \delta(y)$$

If one is interested in radiation far-fields only, then assume:

$$\left\{ d \ , \ \frac{\lambda}{2\pi} \right\} << |\vec{r}|$$

$$\vec{A}(\vec{r}) = \iiint \frac{\mu_o}{4\pi} \frac{\vec{J}(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|} e^{-jk|\vec{r}-\vec{r}\,'|}\, dv' = \hat{z}\, \frac{\mu_o}{4\pi} \int_{-\infty}^{\infty} \frac{I(z')}{|\vec{r} - \hat{z}\, z'|} e^{-jk|\vec{r}-\hat{z}\, z'|}\, dz'$$

$$\vec{A}_{ff}(\vec{r}) \approx \hat{z}\, \frac{\mu_o}{4\pi\, r} \int_{-\infty}^{\infty} I(z')\, e^{-jk(r - \hat{r}.\hat{z}\, z')}\, dz'$$

Far-field approximation - also called the Fraunhoffer approximation

$$= \hat{z}\, \frac{\mu_o}{4\pi\, r} e^{-jkr} \int_{-d/2}^{d/2} I(z')\, e^{jkz'\cos(\theta)}\, dz'$$

Recall that: $\hat{r} = \hat{x}\, \sin(\theta)\cos(\phi) + \hat{y}\, \sin(\theta)\sin(\phi) + \hat{z}\, \cos(\theta)$

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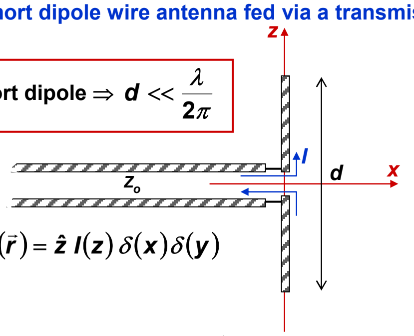
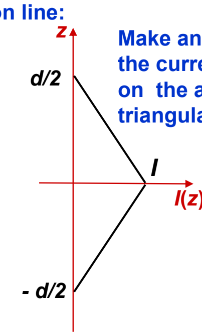
Center Fed Short Dipole Wire Antenna - I

Short dipole wire antenna fed via a transmission line:

Short dipole $\Rightarrow d << \dfrac{\lambda}{2\pi}$



$$\vec{J}(\vec{r}) = \hat{z}\, I(z)\, \delta(x)\delta(y)$$



Make an assumption for the current distribution on the antenna – a triangular distribution

$$\vec{A}_{ff}(\vec{r}) = \hat{z}\,\frac{\mu_o}{4\pi\, r} e^{-j k r} \int_{-d/2}^{d/2} I(z')\, e^{j k z' \cos(\theta)}\, dz'$$

$$\approx \hat{z}\,\frac{\mu_o}{4\pi\, r} e^{-j k r} \int_{-d/2}^{d/2} I(z')\, dz' \quad \rightarrow \quad \left\{ \text{Since: } |z'| < d << \frac{\lambda}{2\pi} \right.$$

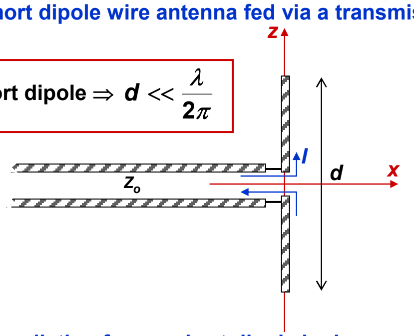
$$= \hat{z}\,\frac{\mu_o\, I d_{eff}}{4\pi\, r} e^{-j k r} \quad \rightarrow \quad \left\{ \text{A Hertzian-dipole-like solution} \right.$$

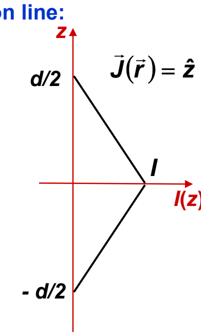
Where: $I\, d_{eff} = \displaystyle\int_{-d/2}^{d/2} I(z')\, dz' = I\,\frac{d}{2} \quad \Rightarrow \quad d_{eff} = \frac{d}{2}$

Center Fed Short Dipole Wire Antenna - II

Short dipole wire antenna fed via a transmission line:

Short dipole $\Rightarrow d << \dfrac{\lambda}{2\pi}$





$$\vec{J}(\vec{r}) = \hat{z}\, I(z)\, \delta(x)\delta(y)$$

The radiation from a short dipole looks like that from a Hertzian dipole except that d is replaced by d_{eff}

$$\vec{E}_{ff}(\vec{r}) = \hat{\theta}\,\frac{j\,\eta_o\, k\, I d_{eff}}{4\pi\, r} \sin(\theta)\, e^{-j k r}$$

$$\vec{H}_{ff}(\vec{r}) = \hat{\phi}\,\frac{j\, k\, I d_{eff}}{4\pi\, r} \sin(\theta)\, e^{-j k r}$$

$$\left\langle \vec{S}(\vec{r},t) \right\rangle = \frac{1}{2} \mathrm{Re}\left\{ \vec{E}_{ff}(\vec{r}) \times \vec{H}_{ff}^{\,*}(\vec{r}) \right\}$$

$$= \hat{r}\,\frac{\eta_o}{2} \left| \frac{k\, I d_{eff}}{4\pi\, r} \right|^2 \sin^2(\theta)$$

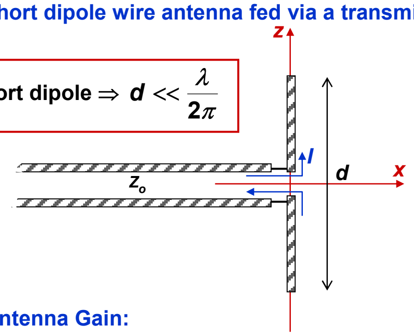
$$P_{rad} = \int_0^{2\pi}\!\!\int_0^{\pi} \left\langle \vec{S}(\vec{r},t) \right\rangle . \hat{r}\;\; r^2 \sin(\theta)\, d\theta\, d\phi$$

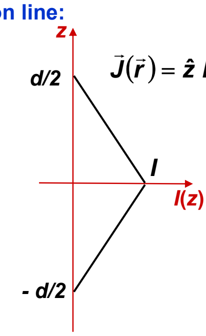
$$= \frac{\eta_o}{12\pi} \left| k\, I d_{eff} \right|^2$$

Center Fed Short Dipole Wire Antenna - III

Short dipole wire antenna fed via a transmission line:

Short dipole $\Rightarrow d << \dfrac{\lambda}{2\pi}$





$$\vec{J}(\vec{r}) = \hat{z}\, I(z)\,\delta(x)\,\delta(y)$$

Antenna Gain:

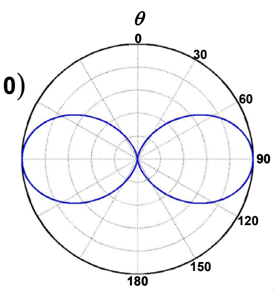
For a short dipole the gain is:

$$G(\theta,\phi) = \frac{\langle \vec{S}(\vec{r},t)\rangle . \hat{r}}{P_{rad}/4\pi r^2} = \frac{3}{2}\sin^2(\theta)$$

Antenna Radiation Pattern:

$$p(\theta,\phi) = \frac{G(\theta,\phi)}{G|_{max}} = \sin^2(\theta)$$

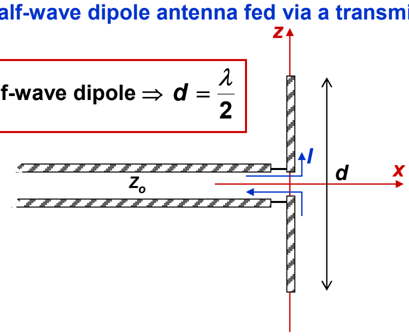
$p(\theta,\phi=0)$

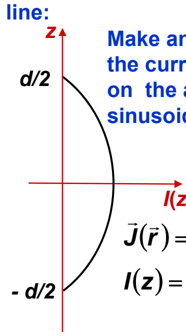


Center Fed Half-Wave Dipole Wire Antenna - I

Half-wave dipole antenna fed via a transmission line:

Half-wave dipole $\Rightarrow d = \dfrac{\lambda}{2}$





Make an assumption for the current distribution on the antenna – a sinusoidal distribution

$$\vec{J}(\vec{r}) = \hat{z}\, I(z)\,\delta(x)\,\delta(y)$$

$$I(z) = I\cos(k z)$$

$$\vec{A}_{ff}(\vec{r}) = \hat{z}\,\frac{\mu_o}{4\pi r} e^{-j k r} \int_{-d/2}^{d/2} I(z')\, e^{j k z' \cos(\theta)}\, dz'$$

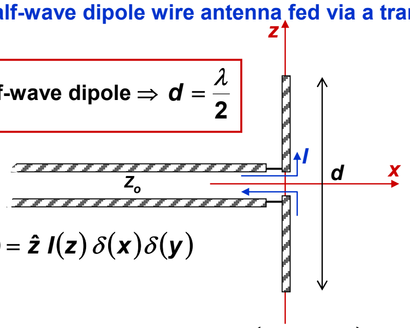
$$\approx \hat{z}\,\frac{\mu_o}{4\pi r} e^{-j k r} \int_{-\lambda/4}^{\lambda/4} I\cos(k z')\, e^{j k z' \cos(\theta)} dz'$$

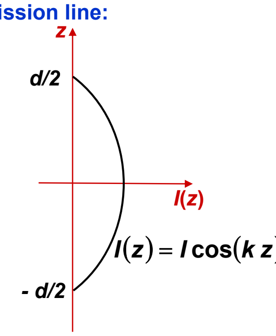
$$= \hat{z}\,\frac{\mu_o I}{2 k \pi r} e^{-j k r} \frac{\cos\left(\frac{\pi}{2}\cos(\theta)\right)}{\sin^2(\theta)}$$

Center Fed Half-Wave Dipole Wire Antenna - II

Half-wave dipole wire antenna fed via a transmission line:

Half-wave dipole $\Rightarrow d = \frac{\lambda}{2}$





$$I(z) = I\cos(kz)$$

$$\vec{J}(\vec{r}) = \hat{z}\, I(z)\, \delta(x)\delta(y)$$

$$\vec{A}_{ff}(\vec{r}) = \hat{z}\, \frac{\mu_o I}{2k\pi r} e^{-jkr} \frac{\cos\left(\frac{\pi}{2}\cos(\theta)\right)}{\sin^2(\theta)}$$

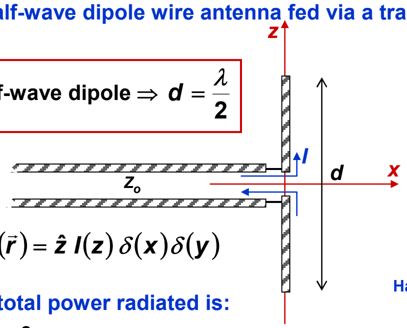
This implies:

$$\vec{E}_{ff}(\vec{r}) = \hat{\theta}\, \frac{j\eta_o I}{2\pi r} e^{-jkr} \frac{\cos\left(\frac{\pi}{2}\cos(\theta)\right)}{\sin(\theta)} \qquad \vec{H}_{ff}(\vec{r}) = \hat{\phi}\, \frac{jI}{2\pi r} e^{-jkr} \frac{\cos\left(\frac{\pi}{2}\cos(\theta)\right)}{\sin(\theta)}$$

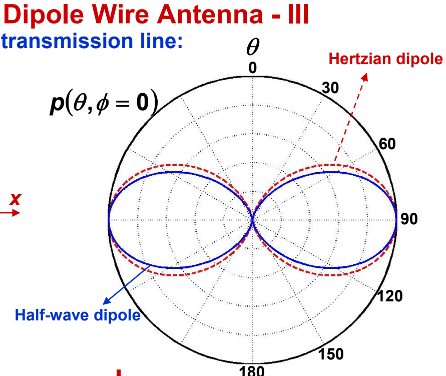
Center Fed Half-Wave Dipole Wire Antenna - III

Half-wave dipole wire antenna fed via a transmission line:

Half-wave dipole $\Rightarrow d = \frac{\lambda}{2}$



$$\vec{J}(\vec{r}) = \hat{z}\, I(z)\, \delta(x)\delta(y)$$



This total power radiated is:

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} \left\langle \vec{S}(\vec{r},t)\right\rangle . \hat{r}\;\; r^2 \sin(\theta) d\theta\, d\phi$$

$$= \int_0^{2\pi}\int_0^{\pi} \frac{\eta_o}{2}\left|\frac{I}{2\pi r}\right|^2 \frac{\cos^2\left(\frac{\pi}{2}\cos(\theta)\right)}{\sin^2(\theta)} r^2 \sin(\theta) d\theta\, d\phi$$

$$\approx \frac{1.22\,\eta_o}{4\pi}|I|^2$$

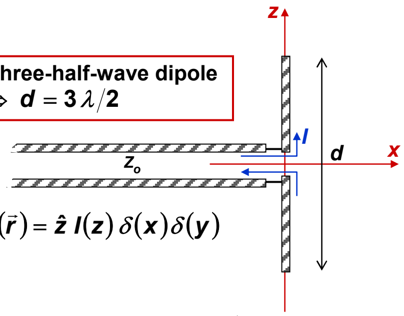
$$R_{rad} = \frac{P_{rad}}{|I|^2/2} \approx 73\,\Omega$$

$$G(\theta,\phi) \approx 1.64\, \frac{\cos^2\left(\frac{\pi}{2}\cos(\theta)\right)}{\sin^2(\theta)}$$

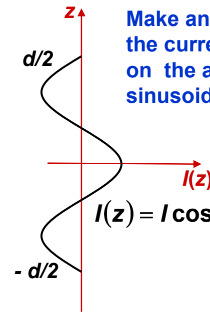
$$p(\theta,\phi) = \frac{\cos^2\left(\frac{\pi}{2}\cos(\theta)\right)}{\sin^2(\theta)}$$

Center Fed Three-Half-Wave Dipole Wire Antenna - I

Three-half-wave dipole
$\Rightarrow d = 3\lambda/2$



$$\vec{J}(\vec{r}) = \hat{z}\, I(z)\, \delta(x)\delta(y)$$



$$I(z) = I\cos(k\,z)$$

Make an assumption for the current distribution on the antenna – a sinusoidal distribution

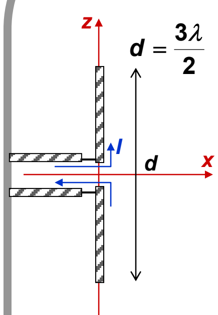
$$\vec{A}_{ff}(\vec{r}) = \hat{z}\,\frac{\mu_o}{4\pi\,r}e^{-j\,k\,r}\int_{-d/2}^{d/2} I(z')\,e^{j\,k\,z'\cos(\theta)}\,dz'$$

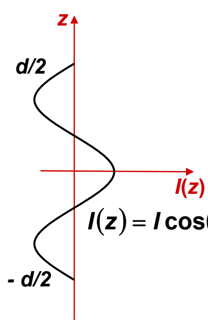
$$\approx \hat{z}\,\frac{\mu_o}{4\pi\,r}e^{-j\,k\,r}\int_{-3\lambda/4}^{3\lambda/4} I\cos(k\,z')\,e^{j\,k\,z'\cos(\theta)}dz'$$

$$= -\hat{z}\,\frac{\mu_o\,I}{2\,k\,\pi\,r}e^{-j\,k\,r}\,\frac{\cos\left(\frac{3\pi}{2}\cos(\theta)\right)}{\sin^2(\theta)}$$

Center Fed Three-Half-Wave Dipole Wire Antenna - II

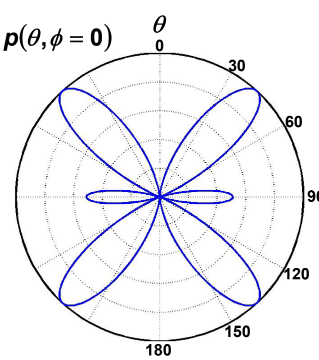
$$d = \frac{3\lambda}{2}$$





$$I(z) = I\cos(k\,z)$$

$p(\theta, \phi = 0)$



$$\vec{E}_{ff}(\vec{r}) = -\hat{\theta}\,\frac{j\,\eta_o\,I}{2\pi\,r}e^{-j\,k\,r}\,\frac{\cos\left(\frac{3\pi}{2}\cos(\theta)\right)}{\sin(\theta)}$$

$$\vec{H}_{ff}(\vec{r}) = -\hat{\phi}\,\frac{j\,I}{2\pi\,r}e^{-j\,k\,r}\,\frac{\cos\left(\frac{3\pi}{2}\cos(\theta)\right)}{\sin(\theta)}$$

$$p(\theta,\phi) \propto \frac{\cos^2\left(\frac{3\pi}{2}\cos(\theta)\right)}{\sin^2(\theta)}$$

Home Made Dipole Antennas

A 1-5 GHz home-made dipole antenna for Wireless LAN with a co-axial SMA RF feed

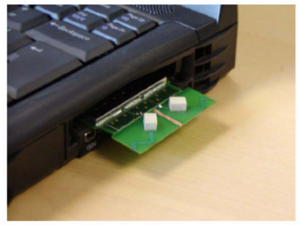
Buddipole ™

A 16 ft dipole for 1-50 MHz radio

Radars for Upper Atmosphere Research

49.92 MHz incoherent scatter radar at the Peru Observatory
The radar has an array of 18,432 half-wave dipoles !!

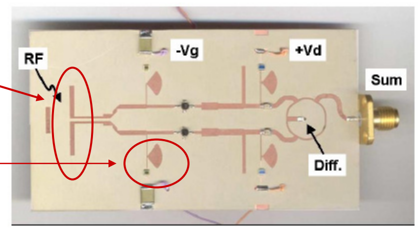
Antennas for Mobile Consumer Products



A PCMCIA card antenna with two crossed short dipoles – shown with the cover removed (for 2-5 GHz)

A short dipole antenna integrated with a low noise amplifier on a PC board for mobile receivers (4-8 GHz)

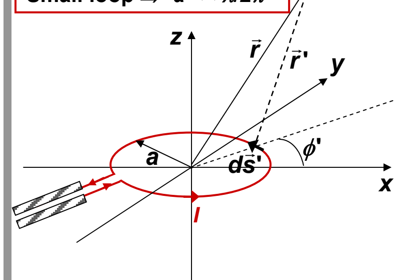
Radial stub tuners for impedance matching



Small Wire Loop Antenna – A Magnetic Dipole Radiator

Consider a small loop of wire carrying time-varying current:

Small loop $\Rightarrow$ $a << \lambda/2\pi$



$$\vec{A}(\vec{r}) = \iiint \frac{\mu_o}{4\pi} \frac{\vec{J}(\vec{r}')}{|\vec{r}-\vec{r}'|} e^{-jk|\vec{r}-\vec{r}'|} \, dv'$$

$$\vec{A}_{ff}(\vec{r}) = \frac{\mu_o}{4\pi r} \int_0^{2\pi} \hat{\phi}' \, I \, e^{-jk(r-\hat{r}.\vec{r}')} \, a d\phi'$$

Note that:

$$\hat{\phi}' = -\hat{x}\sin(\phi') + \hat{y}\cos(\phi')$$

$$\vec{r}' = \hat{x}\, a\cos(\phi') + \hat{y}\, a\sin(\phi')$$

$$\hat{r} = \hat{x}\sin(\theta)\cos(\phi) + \hat{y}\sin(\theta)\sin(\phi) + \hat{z}\cos(\theta)$$

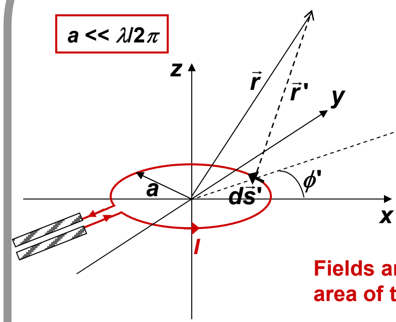
This gives:

$$\vec{A}_{ff}(\vec{r}) = \frac{\mu_o Ia}{4\pi r} e^{-jkr} \int_0^{2\pi} [-\hat{x}\sin(\phi') + \hat{y}\cos(\phi')] \, e^{jka\sin(\theta)[\cos(\phi)\cos(\phi')+\sin(\phi)\sin(\phi')]} \, d\phi'$$

$$\vec{A}_{ff}(\vec{r}) \approx \frac{\mu_o Ia}{4\pi r} e^{-jkr} \int_0^{2\pi} [-\hat{x}\sin(\phi') + \hat{y}\cos(\phi')] \, [1 + jka\sin(\theta)[\cos(\phi)\cos(\phi') + \sin(\phi)\sin(\phi')]] \, d\phi'$$

Small Wire Loop Antenna - II

$a << \lambda/2\pi$



$$\vec{A}_{ff}(\vec{r}) \approx \hat{\phi}\, \frac{j\,\mu_o\, k\, I\left(\pi a^2\right)}{4\pi r} \sin(\theta)\, e^{-j k r}$$

$$\Rightarrow \quad \vec{E}_{ff}(\vec{r}) \approx \hat{\phi}\, \frac{\eta_o\, k^2\, I\left(\pi a^2\right)}{4\pi r} \sin(\theta)\, e^{-j k r}$$

$$\Rightarrow \quad \vec{H}_{ff}(\vec{r}) \approx -\hat{\theta}\, \frac{k^2\, I\left(\pi a^2\right)}{4\pi r} \sin(\theta)\, e^{-j k r}$$

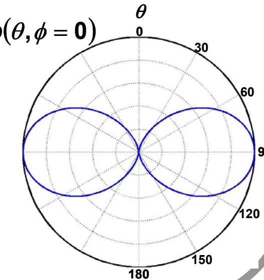
Fields are proportional to the product of the current and the area of the loop

Total power radiated is: $P_{rad} = \frac{\pi\,\eta_o}{12} (k\,a)^4\, |I|^2$

Radiation resistance is: $R_{rad} = \frac{P_{rad}}{|I|^2/2} = \frac{\pi}{6}\eta_o\,(k\,a)^4$

Radiation pattern is: $p(\theta,\phi) = \frac{G(\theta,\phi)}{G|_{max}} = \sin^2(\theta)$

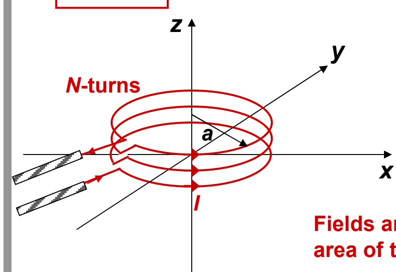
$p(\theta, \phi = 0)$



N-turn Small Wire Loop Antenna

Consider a small loop of wire carrying time-varying current:

$a << \lambda/2\pi$



$$\vec{A}_{ff}(\vec{r}) \approx \hat{\phi}\, \frac{j\,\mu_o\, k\, N\, I\left(\pi a^2\right)}{4\pi r} \sin(\theta)\, e^{-j k r}$$

$$\Rightarrow \quad \vec{E}_{ff}(\vec{r}) \approx \hat{\phi}\, \frac{\eta_o\, k^2\, N\, I\left(\pi a^2\right)}{4\pi r} \sin(\theta)\, e^{-j k r}$$

$$\Rightarrow \quad \vec{H}_{ff}(\vec{r}) \approx -\hat{\theta}\, \frac{k^2\, N\, I\left(\pi a^2\right)}{4\pi r} \sin(\theta)\, e^{-j k r}$$

Fields are proportional to the product of the current and the area of the loop

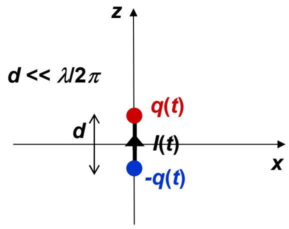
Total power radiated is: $P_{rad} = \frac{\pi\,\eta_o}{12} (k\,a)^4\, |N\,I|^2$

Radiation resistance is: $R_{rad} = \frac{P}{|I|^2/2} = \frac{\pi}{6}\eta_o\, N^2 (k\,a)^4$

Radiation pattern is: $p(\theta,\phi) = \frac{G(\theta,\phi)}{G|_{max}} = \sin^2(\theta)$

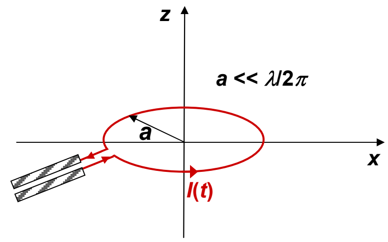
It is easier to obtain larger radiation resistances with small loop antennas (containing many turns) than with short dipole antennas of the same size

Electric Dipole Radiators Vs Magnetic Dipole Radiators



$$\vec{E}_{nf}(\vec{r},t) = \frac{q(t)d}{4\pi\,\varepsilon_o\,r^3}\left[\,\hat{r}\;2\cos(\theta) + \hat{\theta}\;\sin(\theta)\,\right]$$

The electric near-field looks like that of an electric dipole



The magnetic near-field looks like that of a magnetic dipole

$$\vec{H}_{nf}(\vec{r},t) = \frac{I(t)\,\pi\,a^2}{4\pi\;r^3}\left[\,\hat{r}\;2\cos(\theta) + \hat{\theta}\;\sin(\theta)\,\right]$$

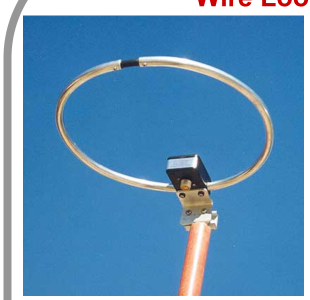
$$\vec{E}_{ff}(\vec{r}) = \hat{\theta}\,\frac{j\,\eta_o\,k\,Id}{4\pi\,r}\sin(\theta)\,e^{-j\,k\,r}$$

$$\vec{H}_{ff}(\vec{r}) = \hat{\phi}\,\frac{j\,k\,Id}{4\pi\,r}\sin(\theta)\,e^{-j\,k\,r}$$

$$\vec{E}_{ff}(\vec{r}) = \hat{\phi}\,\frac{\eta_o\,k^2\,I\left(\pi\,a^2\right)}{4\pi\,r}\sin(\theta)\,e^{-j\,k\,r}$$

$$\vec{H}_{ff}(\vec{r}) = -\hat{\theta}\,\frac{k^2\,I\left(\pi\,a^2\right)}{4\pi\,r}\sin(\theta)\,e^{-j\,k\,r}$$

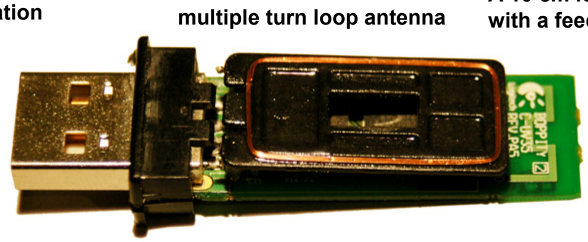
Wire Loop Antennas in Consumer Products



A 2 m loop antenna for 1-30 MHz operation

A 30 inch home made multiple turn loop antenna

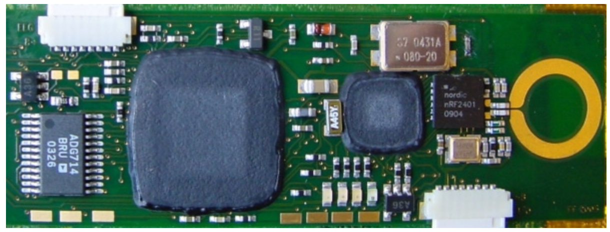
A 10 cm loop antenna with a feed



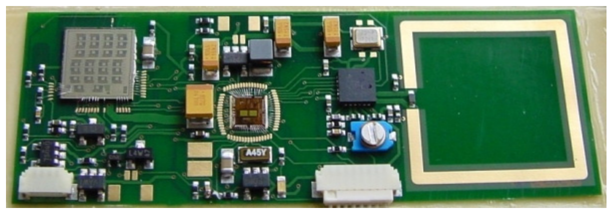
LOGITECH BLUETOOTH DONGLE WITH COIL ANTENNA MOUNTED

Wire Loop Antennas in Medical Devices - II

A flexible "band-aid" chip for wireless EEG (Electroencephalography) measurements at 2.4 GHz with a loop antenna



A flexible "band-aid" chip for wireless EMG (Electromyography) measurements at 433 MHz with a loop antenna



Wire Loop Antennas for Everybody - III

A portable loop antenna for 5-10 MHz operation on somebody's van

A home made loop antenna in somebody's backyard

Mine is bigger – says this guy!