

Lecture 23: Addition

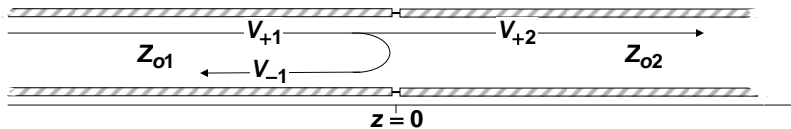
Multilayer Structures – Non-Normal Incidence

In this lecture you will learn:

- Multilayer structures: non-normal incidence

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Waves at Interfaces and Transmission Lines



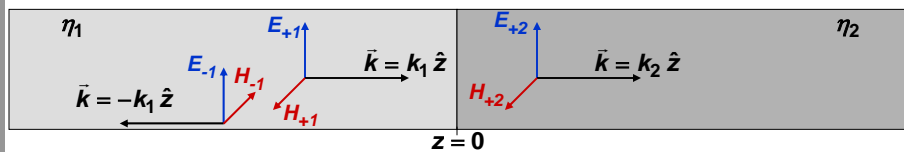
$$V(z)_{z<0} = V_{+1} e^{-jk_1 z} + V_{-1} e^{+jk_1 z}$$

$$V(z)_{z>0} = V_{+2} e^{-jk_2 z}$$

Boundary conditions:

$$(1) \Rightarrow V_{+1} + V_{-1} = V_{+2}$$

$$(2) \Rightarrow \frac{V_{+1}}{Z_{o1}} - \frac{V_{-1}}{Z_{o1}} = \frac{V_{+2}}{Z_{o2}} \quad \longrightarrow \quad \Gamma = \frac{V_{-1}}{V_{+1}} = \frac{Z_{o2}/Z_{o1} - 1}{Z_{o2}/Z_{o1} + 1}$$



$$\vec{E}(z)_{z<0} = \hat{x} E_{+1} e^{-jk_1 z} + \hat{x} E_{-1} e^{+jk_1 z}$$

$$\vec{E}(z)_{z>0} = \hat{x} E_{+2} e^{-jk_2 z}$$

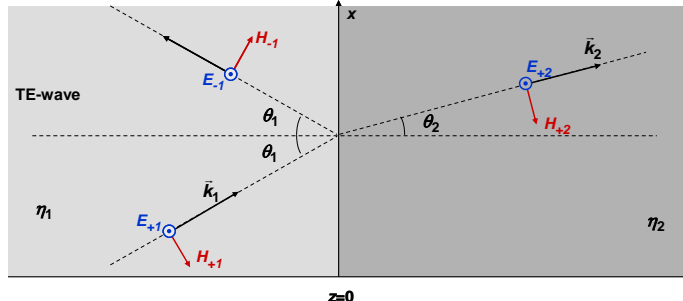
Boundary conditions:

$$(1) \Rightarrow E_{+1} + E_{-1} = E_{+2}$$

$$(2) \Rightarrow \frac{E_{+1}}{\eta_1} - \frac{E_{-1}}{\eta_1} = \frac{E_{+2}}{\eta_2} \quad \longrightarrow \quad \Gamma = \frac{E_{-1}}{E_{+1}} = \frac{\eta_2/\eta_1 - 1}{\eta_2/\eta_1 + 1}$$

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Waves at Interfaces: TE Wave - I



$$\vec{E}(\vec{r})|_{z<0} = \hat{y} E_{+1} e^{-j(\bar{k}_{1x} x + \bar{k}_{1z} z)} + \hat{y} E_{-1} e^{-j(\bar{k}_{1x} x - \bar{k}_{1z} z)}$$

$$\vec{E}(\vec{r})|_{z>0} = \hat{y} E_{+2} e^{-j(k_{2x} x + k_{2z} z)}$$

Boundary Conditions:

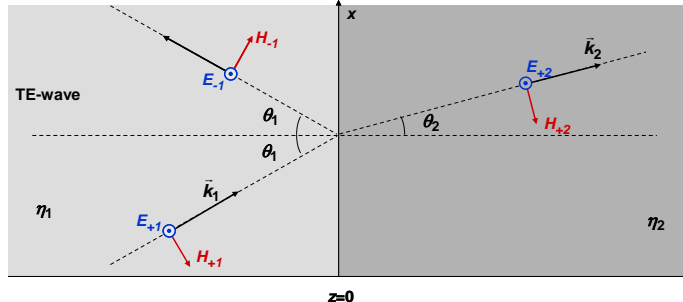
$$(1) \Rightarrow E_{+1} + E_{-1} = E_{+2}$$

$$(2) \Rightarrow \left[\frac{E_{+1}}{\eta_1/\cos(\theta_1)} - \frac{E_{-1}}{\eta_1/\cos(\theta_1)} \right] = \frac{E_{+2}}{\eta_2/\cos(\theta_2)}$$

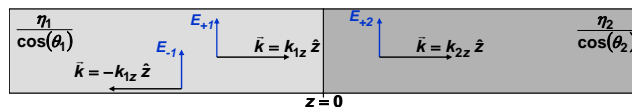
$$\left. \begin{aligned} T = \frac{E_{+2}}{E_{+1}} &= \frac{2 \eta_2 / \cos(\theta_2)}{\eta_1 / \cos(\theta_1) + 1} \\ \Gamma = \frac{E_{-1}}{E_{+1}} &= \frac{\eta_2 / \cos(\theta_2) - \eta_1 / \cos(\theta_1)}{\eta_2 / \cos(\theta_2) + \eta_1 / \cos(\theta_1)} \end{aligned} \right\}$$

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Waves at Interfaces: TE Wave - II



One may replace the above problem with a "dummy" normal incidence problem:



And then calculate the reflection and transmission coefficients for the E-field:

$$\Gamma = \frac{E_{-1}}{E_{+1}} = \frac{\frac{\eta_2 / \cos(\theta_2) - 1}{\eta_1 / \cos(\theta_1)}}{\frac{\eta_2 / \cos(\theta_2) + 1}{\eta_1 / \cos(\theta_1)}}$$

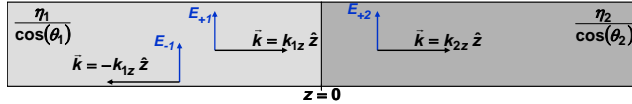
$$T = \frac{E_{+2}}{E_{+1}} = \frac{2 \eta_2 / \cos(\theta_2)}{\frac{\eta_2 / \cos(\theta_2) + 1}{\eta_1 / \cos(\theta_1)}}$$

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Waves at Interfaces: TE Wave - III

In the “dummy” normal incidence problem:

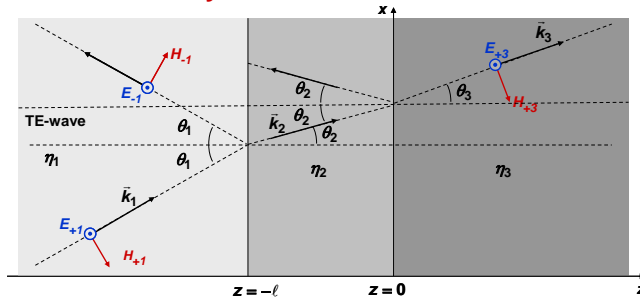
- The wavevectors are taken to be the z-component of the actual wavevectors in each medium
- The impedances in each medium are taken to be the actual impedances divided by the cosines of the angles of propagation w.r.t. the z-axis



$$\Gamma = \frac{E_{-1}}{E_{+1}} = \frac{\frac{\eta_2/\cos(\theta_2)}{\eta_1/\cos(\theta_1)} - 1}{\frac{\eta_2/\cos(\theta_2)}{\eta_1/\cos(\theta_1)} + 1} \quad T = \frac{E_{+2}}{E_{+1}} = \frac{2 \frac{\eta_2/\cos(\theta_2)}{\eta_1/\cos(\theta_1)}}{\frac{\eta_2/\cos(\theta_2)}{\eta_1/\cos(\theta_1)} + 1}$$

Warning: The “dummy” normal incidence problem is only meant to be used to calculate reflection and transmission coefficients – don’t use this framework for anything else

Tri-layer Structure: TE Wave - I



$$\vec{E}(\vec{r})|_{z < -l} = \hat{y} E_{+1} e^{-j(\bar{k}_{1x} x + \bar{k}_{1z}(z+l))} + \hat{y} E_{-1} e^{-j(\bar{k}_{1x} x - \bar{k}_{1z}(z+l))}$$

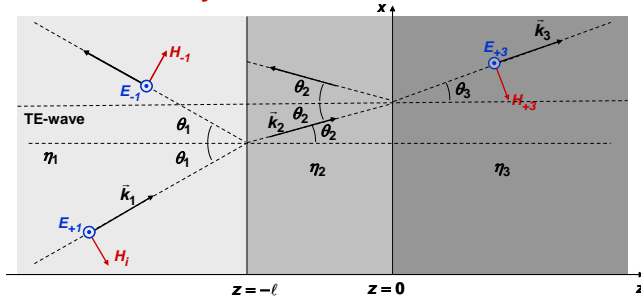
$$\vec{E}(\vec{r})|_{-l < z < 0} = \hat{y} E_{+2} e^{-j(\bar{k}_{2x} x + \bar{k}_{2z} z)} + \hat{y} E_{-2} e^{-j(\bar{k}_{2x} x - \bar{k}_{2z} z)}$$

$$\vec{E}(\vec{r})|_{z > 0} = \hat{y} E_{+3} e^{-j(k_{3x} x + k_{3z} z)}$$

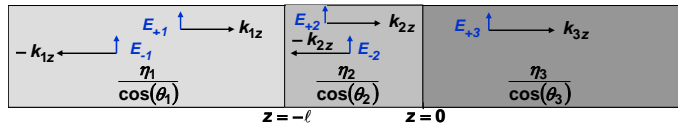
How does one solve a problem like this?

$$\Gamma = \frac{E_{-1}}{E_{+1}} = ?$$

Tri-layer Structure: TE Wave - II

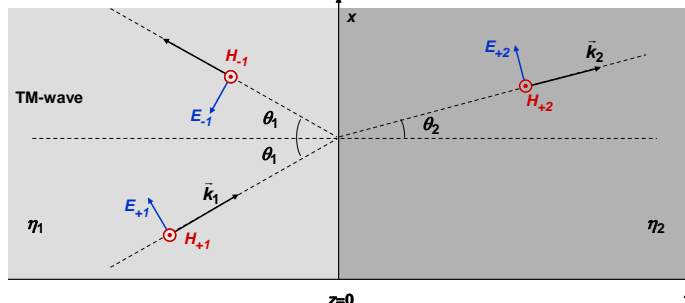


Replace the above problem with a "dummy" normal incidence problem:



And then use the usual methods to solve it (impedance transformations, Smith chart, etc)

Waves at Interfaces: TM Wave - I



Work with the E-field component parallel to the media interface (i.e. the x-component)

$$E_x(\vec{r})_{z < 0} = E_{+1x} e^{-j(\bar{k}_{1x} x + \bar{k}_{1z} z)} + E_{-1x} e^{-j(\bar{k}_{1x} x - \bar{k}_{1z} z)}$$

$$E_x(\vec{r})_{z > 0} = E_{+2x} e^{-j(k_{2x} x + k_{2z} z)}$$

Boundary Conditions:

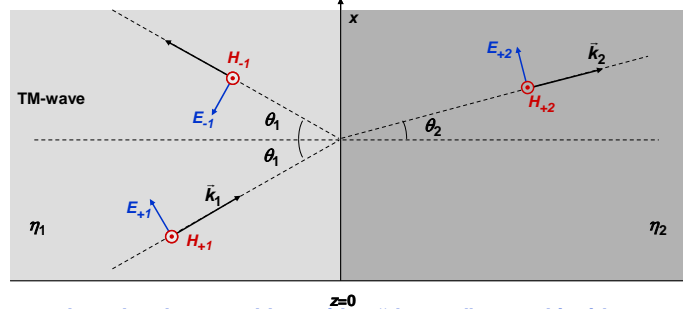
$$(1) \Rightarrow E_{+1x} + E_{-1x} = E_{+2x}$$

$$(2) \Rightarrow \left[\frac{E_{+1x}}{\eta_1 \cos(\theta_1)} - \frac{E_{-1x}}{\eta_1 \cos(\theta_1)} \right] = \frac{E_{+2x}}{\eta_2 \cos(\theta_2)}$$

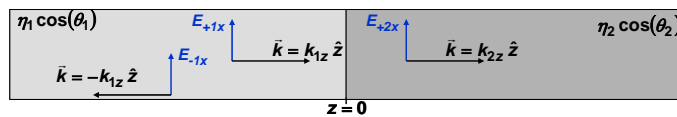
$$\Gamma = \frac{E_{-1x}}{E_{+1x}} = \frac{\frac{\eta_2 \cos(\theta_2)}{\eta_1 \cos(\theta_1)} - 1}{\frac{\eta_2 \cos(\theta_2)}{\eta_1 \cos(\theta_1)} + 1}$$

$$T = \frac{E_{+2x}}{E_{+1x}} = \frac{2 \frac{\eta_2 \cos(\theta_2)}{\eta_1 \cos(\theta_1)}}{\frac{\eta_2 \cos(\theta_2)}{\eta_1 \cos(\theta_1)} + 1}$$

Waves at Interfaces: TM Wave - II



One may replace the above problem with a “dummy” normal incidence problem:

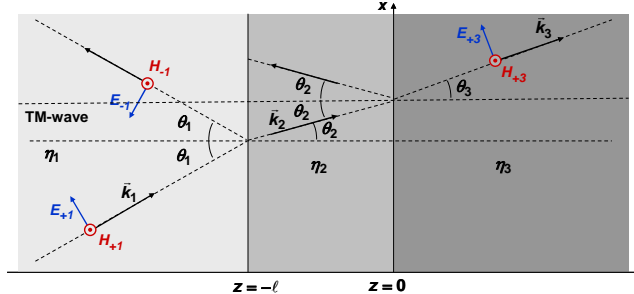


And then calculate the reflection and transmission coefficients for the x-component of the E-field:

$$\Gamma = \frac{E_{-1x}}{E_{+1x}} = \frac{\frac{\eta_2 \cos(\theta_2)}{\eta_1 \cos(\theta_1)} - 1}{\frac{\eta_2 \cos(\theta_2)}{\eta_1 \cos(\theta_1)} + 1} \quad T = \frac{E_{+2x}}{E_{+1x}} = \frac{2 \frac{\eta_2 \cos(\theta_2)}{\eta_1 \cos(\theta_1)}}{\frac{\eta_2 \cos(\theta_2)}{\eta_1 \cos(\theta_1)} + 1}$$

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Tri-layer Structure: TM Wave - I



Work with the E-field component parallel to the media interface (i.e. the x-component)

$$E_x(\vec{r})_{z < -l} = E_{+1x} e^{-j(\bar{k}_{1x} x + \bar{k}_{1z}(z+l))} + E_{-1x} e^{-j(\bar{k}_{1x} x - \bar{k}_{1z}(z+l))}$$

$$E_x(\vec{r})_{-l < z < 0} = E_{+2x} e^{-j(\bar{k}_{2x} x + \bar{k}_{2z} z)} + E_{-2x} e^{-j(\bar{k}_{2x} x - \bar{k}_{2z} z)}$$

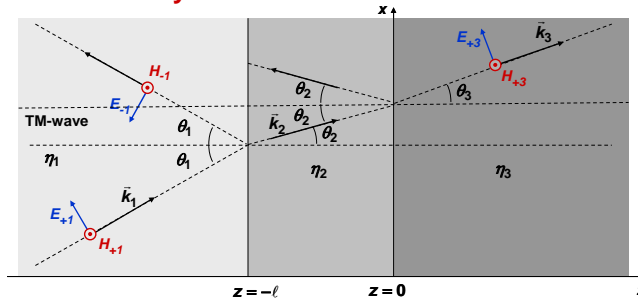
$$E_x(\vec{r})_{z > 0} = E_{+3x} e^{-j(k_{3x} x + k_{3z} z)}$$

How does one solve a problem like this?

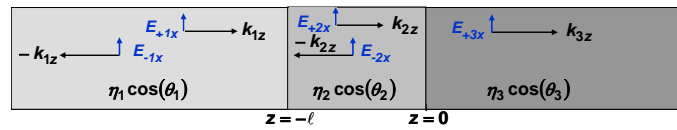
$$\Gamma = \frac{E_{-1x}}{E_{+1x}} = ?$$

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Tri-layer Structure: TM Wave - II



Replace the above problem with a "dummy" normal incidence problem:



And then use the usual methods to solve it (impedance transformations, Smith chart, etc)