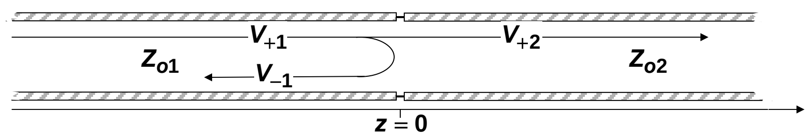
Lecture 23

Multilayer Structures

In this lecture you will learn:

- Multilayer structures
- Dielectric anti-reflection (AR) coatings
- Dielectric high-reflection (HR) coatings
- Photonic Band-Gap Structures

Transmission Line Junctions and Discontinuities - I



Transmission line discontinuities generate reflections

$$V(z)\big|_{z<0} = V_{+1}\, e^{-j k_1 z} + V_{-1}\, e^{+j k_1 z} \qquad\qquad V(z)\big|_{z>0} = V_{+2}\, e^{-j k_2 z}$$

Boundary conditions:

1) Continuity of voltage at z=0:

$$V_{+1} + V_{-1} = V_{+2}$$

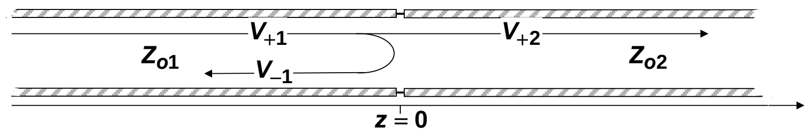
1) Continuity of current at z=0:

$$\frac{V_{+1}}{Z_{o1}} - \frac{V_{-1}}{Z_{o1}} = \frac{V_{+2}}{Z_{o2}}$$

$$\longrightarrow \quad \Gamma = \frac{V_{-1}}{V_{+1}} = \frac{Z_{o2}/Z_{o1} - 1}{Z_{o2}/Z_{o1} + 1}$$

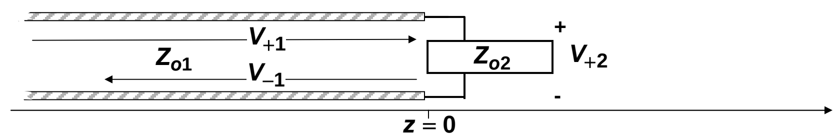
$$T = \frac{V_{+2}}{V_{+1}} = \frac{2\, Z_{o2}/Z_{o1}}{Z_{o2}/Z_{o1} + 1}$$

Transmission Line Junctions and Discontinuities - II



$$V(z)\big|_{z<0} = V_{+1}\,e^{-j k_1 z} + V_{-1}\,e^{+j k_1 z} \qquad V(z)\big|_{z>0} = V_{+2}\,e^{-j k_2 z}$$

Can also replace the infinite transmission line on the right by a lumped impedance



Which gives:

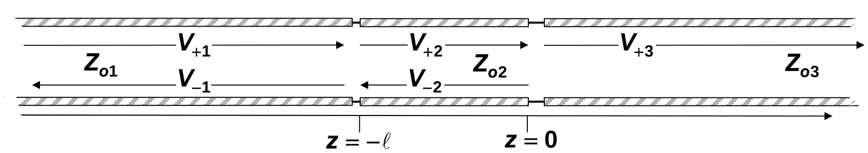
$$\Gamma = \frac{V_{-1}}{V_{+1}} = \frac{Z_{o2}/Z_{o1} - 1}{Z_{o2}/Z_{o1} + 1}$$

$$V_{+2} = V(z=0) = V_{+1} + V_{-1} = V_{+1}(1+\Gamma)$$

$$\Rightarrow T = \frac{V_{+2}}{V_{+1}} = 1 + \Gamma = \frac{2\,Z_{o2}/Z_{o1}}{Z_{o2}/Z_{o1} + 1}$$

Unmatched Transmission Lines - I

Question: How does one solve a problem like this?



$$V(z)\big|_{z<-\ell} = V_{+1}\,e^{-j k_1 (z+\ell)} + V_{-1}\,e^{+j k_1 (z+\ell)}$$

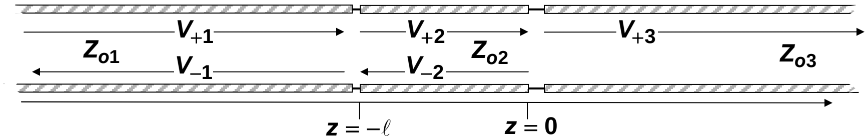
$$V(z)\big|_{-\ell<z<0} = V_{+2}\,e^{-j k_2 z} + V_{-2}\,e^{+j k_2 z}$$

$$V(z)\big|_{z>0} = V_{+3}\,e^{-j k_3 z}$$

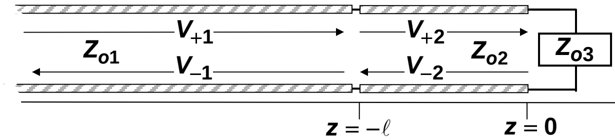
- In each segment (except the right most one), the wave is written such that the phase is zero at the right end of the segment
- In each segment, the phase has the wavevector corresponding to that segment

Unmatched Transmission Lines - II

Question: How does one solve a problem like this?



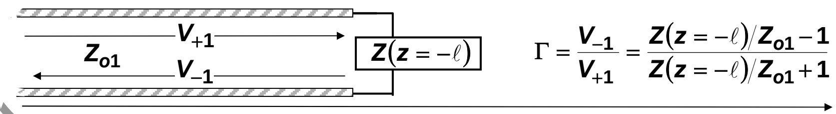
STEP 1: Replace the last line with a lumped equivalent impedance (corresponding to an infinite line)



$$\Gamma = \frac{V_{-2}}{V_{+2}} = \frac{Z_{o3}/Z_{o2} - 1}{Z_{o3}/Z_{o2} + 1}$$

Now calculate the impedance $Z(z=-\ell)$: $Z(z=-\ell) = Z_{o2}\dfrac{1+\Gamma e^{-2jk_2\ell}}{1-\Gamma e^{-2jk_2\ell}}$

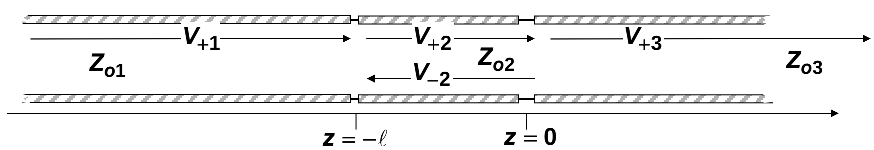
STEP 2: Replace the middle line with the impedance $Z(z=-\ell)$



$$\Gamma = \frac{V_{-1}}{V_{+1}} = \frac{Z(z=-\ell)/Z_{o1} - 1}{Z(z=-\ell)/Z_{o1} + 1}$$

Matching Transmission Lines - I

Question: Is it possible to use a transmission line to perfectly match two dissimilar transmission lines so that there is no reflection?



What is the appropriate impedance Z_{o2}? What is the appropriate length ℓ ?

Use a Quarter-Wave Transformer:

Suppose the length ℓ of the intermediate transmission line is quarter-wavelength

$$\ell = \frac{\lambda_2}{4} \quad \Rightarrow \quad k_2\,\ell = \frac{\pi}{2}$$

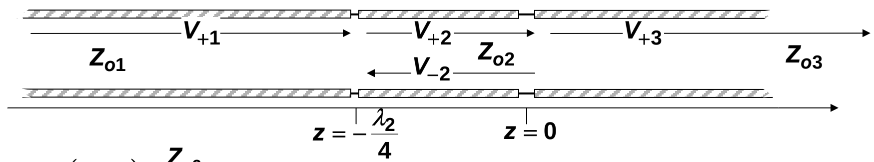
$$Z_n(z) = \frac{Z(z)}{Z_{o2}}$$

$$\Rightarrow \quad Z_n(z=0) = \frac{Z_{o3}}{Z_{o2}}$$

$$\Rightarrow \quad Z_n\left(z = -\frac{\lambda_2}{4}\right) = \frac{1}{Z_n(z=0)}$$

A quarter-wavelength long transmission line inverts the normalized impedance

Matching Transmission Lines - II



$$Z_n(z=0) = \frac{Z_{o3}}{Z_{o2}}$$

$$Z_n\left(z = -\frac{\lambda_2}{4}\right) = \frac{1}{Z_n(z=0)} = \frac{Z_{o2}}{Z_{o3}}$$

The actual impedance at $z = -\lambda_2/4$ is then: $Z\left(z = -\frac{\lambda_2}{4}\right) = Z_{o2}\, Z_n\left(z = -\frac{\lambda_2}{4}\right) = \frac{Z_{o2}^2}{Z_{o3}}$

To have no reflection we need: $Z\left(z = -\frac{\lambda_2}{4}\right) = Z_{o1}$

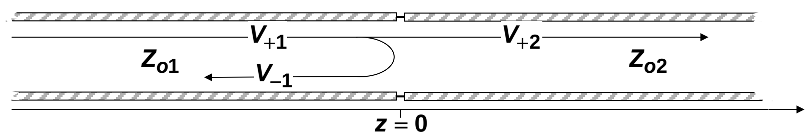
$$\Rightarrow \quad \frac{Z_{o2}^2}{Z_{o3}} = Z_{o1}$$

$$\Rightarrow \quad Z_{o2}^2 = Z_{o1}\, Z_{o3}$$

$$\Rightarrow \quad Z_{o2} = \sqrt{Z_{o1}\, Z_{o3}}$$

The impedance of the quarter-wavelength long transmission line must be the geometric mean of the impedances of the two transmission lines

Waves at Interfaces and Transmission Lines

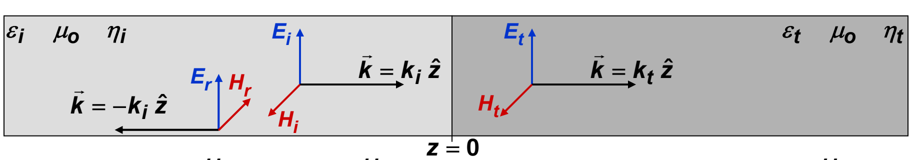


$$V(z)\big|_{z<0} = V_{+1}\, e^{-j k_1 z} + V_{-1}\, e^{+j k_1 z} \qquad V(z)\big|_{z>0} = V_{+2}\, e^{-j k_2 z}$$

Boundary conditions:

$$(1) \quad \Rightarrow \quad V_{+1} + V_{-1} = V_{+2}$$

$$(2) \quad \Rightarrow \quad \frac{V_{+1}}{Z_{o1}} - \frac{V_{-1}}{Z_{o1}} = \frac{V_{+2}}{Z_{o2}} \quad \longrightarrow \quad \Gamma = \frac{V_{-1}}{V_{+1}} = \frac{Z_{o2}/Z_{o1} - 1}{Z_{o2}/Z_{o1} + 1}$$



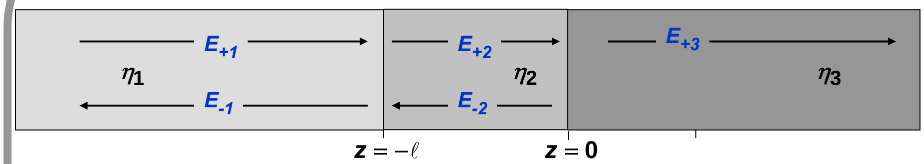
$$\vec{E}(z)\big|_{z<0} = \hat{x}\, E_i\, e^{-j k_i z} + \hat{x}\, E_r\, e^{+j k_i z} \qquad \vec{E}(z)\big|_{z>0} = \hat{x}\, E_t\, e^{-j k_t z}$$

Boundary conditions:

$$(1) \quad \Rightarrow \quad E_i + E_r = E_t$$

$$(2) \quad \Rightarrow \quad \frac{E_i}{\eta_i} - \frac{E_r}{\eta_i} = \frac{E_t}{\eta_t} \quad \longrightarrow \quad \Gamma = \frac{E_r}{E_i} = \frac{\eta_t/\eta_i - 1}{\eta_t/\eta_i + 1}$$

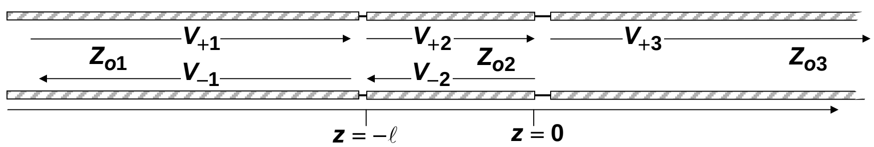
Tri-layer Structure - I



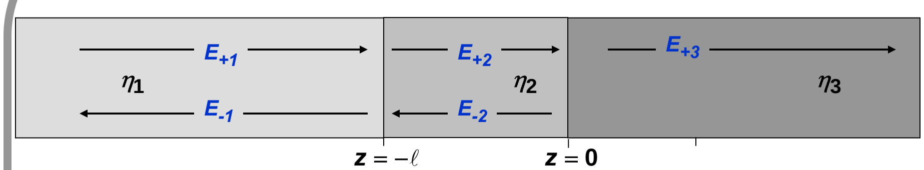
Question: How do we calculate the reflection coefficient for the above structure?

$$\Gamma = \frac{E_{-1}}{E_{+1}} = ?$$

Answer: Use the method employed earlier in the equivalent transmission line problem



Tri-layer Structure - II



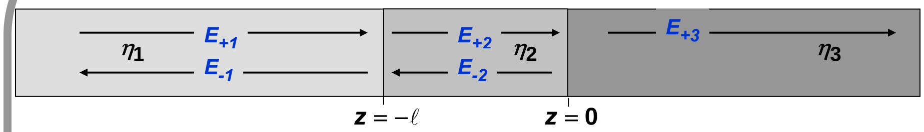
$$E_x(z)\big|_{z<-\ell} = E_{+1}\, e^{-j k_1 (z+\ell)} + E_{-1}\, e^{+j k_1 (z+\ell)}$$

$$E_x(z)\big|_{-\ell<z<0} = E_{+2}\, e^{-j k_2 z} + E_{-2}\, e^{+j k_2 z}$$

$$E_x(z)\big|_{z>0} = E_{+3}\, e^{-j k_3 z}$$

- In each segment (except the right most one), the wave is written such that the phase is zero at the right end of the segment
- In each segment, the phase has the wavevector corresponding to that segment

Tri-layer Structure - III

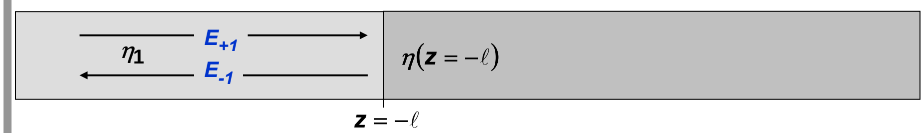


STEP 1: Calculate the reflection at z = 0:

$$\Gamma = \frac{E_{-2}}{E_{+2}} = \frac{\eta_3/\eta_2 - 1}{\eta_3/\eta_2 + 1}$$

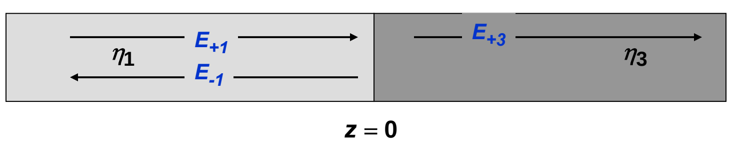
And calculate the effective impedance at z=-ℓ : $\eta(z=-\ell) = \eta_2 \dfrac{1+\Gamma e^{-2jk_2\ell}}{1-\Gamma e^{-2jk_2\ell}}$

STEP 2: Now the problem becomes:



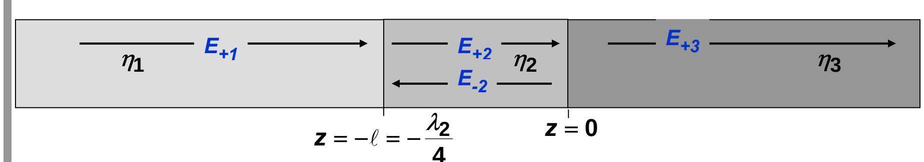
Calculate the reflection coefficient as: $\Gamma = \dfrac{E_{-1}}{E_{+1}} = \dfrac{\eta(z=-\ell)/\eta_1 - 1}{\eta(z=-\hbar)/\eta_1 + 1}$

Dielectric Anti-Reflection (AR) Coatings - I

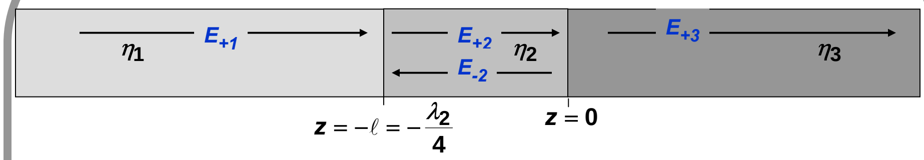


Question: Is it possible to somehow make the reflection coefficient zero?

Answer: Use the quarter-wave transformer concept:



Dielectric Anti-Reflection (AR) Coatings - II



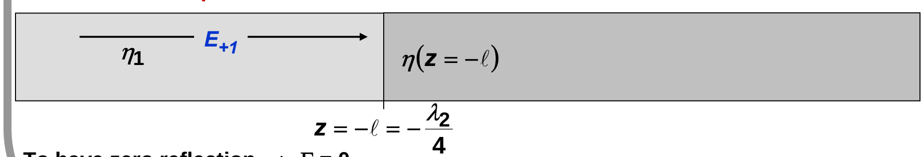
STEP 1: Calculate the normalized impedance at $z = 0$ and at $z = -\lambda_2 / 4$

$$\eta_n(z) = \frac{\eta(z)}{\eta_2} \quad \Rightarrow \quad \eta_n(z=0) = \frac{\eta_3}{\eta_2}$$

$$\Rightarrow \quad \eta_n\left(z = -\frac{\lambda_2}{4}\right) = \frac{1}{\eta_n(z=0)} = \frac{\eta_2}{\eta_3} \longrightarrow$$

A quarter-wavelength long segment inverts the normalized impedance

STEP 2: Now the problem becomes:

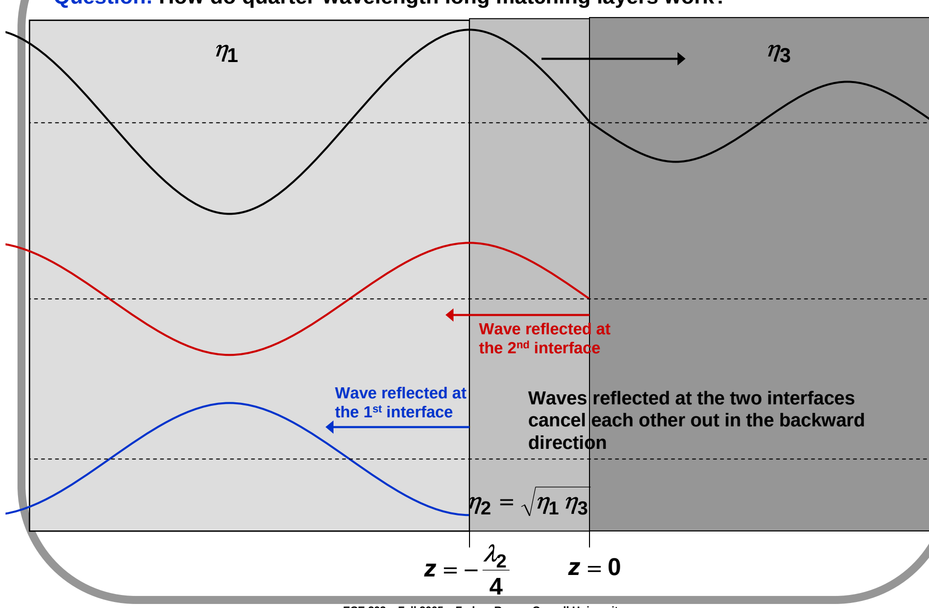


To have zero reflection $\Rightarrow \Gamma = 0$

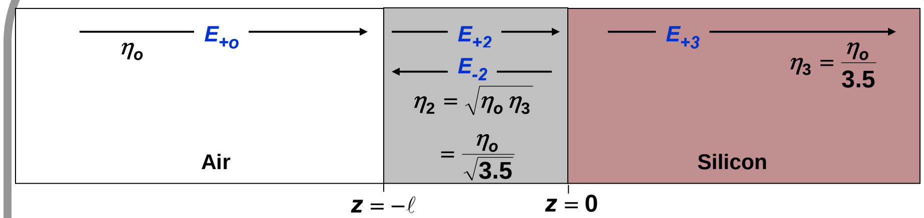
$$\Rightarrow \quad \eta_1 = \eta(z = -\ell) = \eta_2\, \eta_n\left(z = -\frac{\lambda_2}{4}\right) = \frac{\eta_2^2}{\eta_3} \quad \Rightarrow \quad \boxed{\eta_2 = \sqrt{\eta_1 \eta_3}}$$

Dielectric Anti-Reflection (AR) Coatings - III

Question: How do quarter-wavelength long matching layers work?



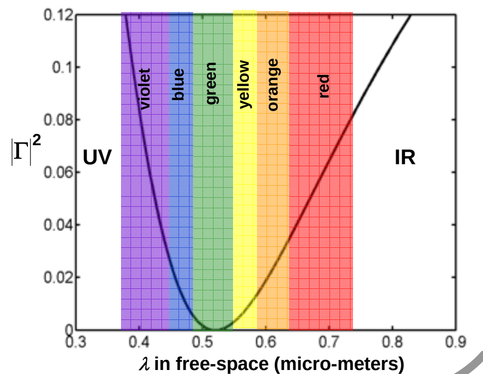
Frequency Dependence of Dielectric AR Coatings: Example



Consider the AR coating on a Silicon photodetector that is designed for detecting green light which has a frequency ω of 3.625x10^{15} rad/sec and a wavelength of 0.52 micro-meter in free-space

$$\ell = \frac{\lambda_{2g}}{4} = \frac{0.52\times10^{-6}/\sqrt{3.5}}{4}\ \text{m}$$

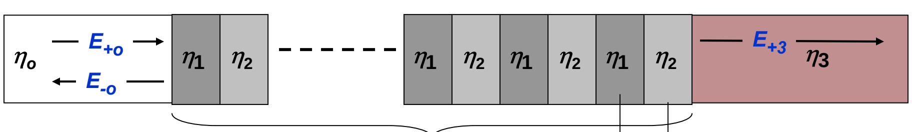
Question: How good will the AR coating work for frequencies different from that of green light (e.g. for blue light, violet light, orange light, etc)



Dielectric High-Reflection (HR) Coatings - I

Question: What if we want to increase the reflectivity?

Answer: A periodic stack of high index-low index layers can be used as an HR coating



- N pairs of (high index – low index) layers
- Total 2N layers
- Each layer is quarter-wavelength thick
- One period has thickness ($\lambda_1/4 + \lambda_2/4$)

Thickness: $\frac{\lambda_2}{4}$

Thickness: $\frac{\lambda_1}{4}$

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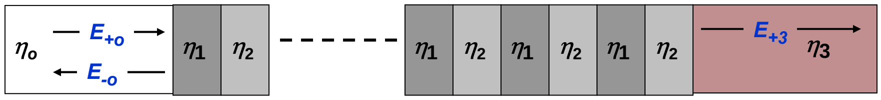
Dielectric High-Reflection (HR) Coatings - II



Start at the right end:

Impedance looking to the right: η_3

Normalized impedance: $\dfrac{\eta_3}{\eta_2}$

Normalized impedance: $\dfrac{\eta_2}{\eta_3}$

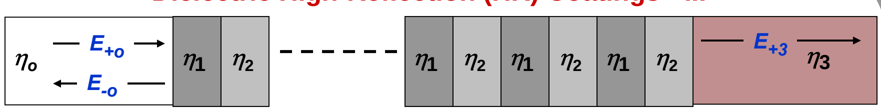
Actual impedance looking to the right: $\dfrac{\eta_2^2}{\eta_3}$

Normalized impedance: $\dfrac{1}{\eta_3}\dfrac{\eta_2^2}{\eta_1}$

Normalized impedance: $\eta_3\dfrac{\eta_1}{\eta_2^2}$

Actual impedance looking to the right: $\eta_3\dfrac{\eta_1^2}{\eta_2^2} = \eta_3\left(\dfrac{\eta_1}{\eta_2}\right)^2$

Dielectric High-Reflection (HR) Coatings - III



Actual impedance looking to the right: $\eta_3\dfrac{\eta_1^2}{\eta_2^2} = \eta_3\left(\dfrac{\eta_1}{\eta_2}\right)^2$

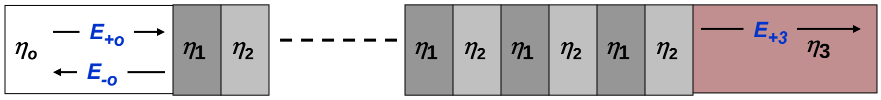
Continuing like this:

Actual impedance looking to the right: $\eta_3\left(\dfrac{\eta_1}{\eta_2}\right)^4$

Actual impedance looking to the right: $\eta_3\left(\dfrac{\eta_1}{\eta_2}\right)^6$

Actual impedance looking to the right: $\eta_3\left(\dfrac{\eta_1}{\eta_2}\right)^{2N}$

Dielectric High-Reflection (HR) Coatings - III



Actual impedance looking to the right: $\eta_3 \left(\frac{\eta_1}{\eta_2}\right)^{2N}$

Reflection Coefficient: $\Gamma = \frac{E_{-o}}{E_{+o}} = \frac{\frac{\eta_3}{\eta_o}\left(\frac{\eta_1}{\eta_2}\right)^{2N} - 1}{\frac{\eta_3}{\eta_o}\left(\frac{\eta_1}{\eta_2}\right)^{2N} + 1}$

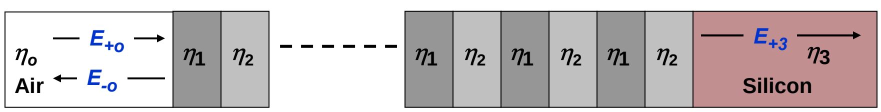
if $\eta_1 > \eta_2 \Rightarrow \Gamma \approx +1$

if $\eta_1 < \eta_2 \Rightarrow \Gamma \approx -1$

Magnitude of Γ can be made arbitrarily close to unity by:

i) **Choosing a large number *N* of pairs of high index-low index layers**
ii) **Choosing the difference between η_1 and η_2 to be large**

Frequency Dependence of HR Coatings: Example

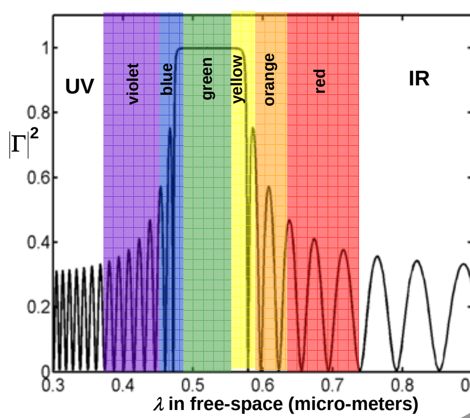


$N = 20$ $\quad \eta_2 = \frac{\eta_o}{1.5}$ $\quad \eta_1 = \frac{\eta_o}{3.5}$ $\quad \eta_3 = \frac{\eta_o}{3.5}$

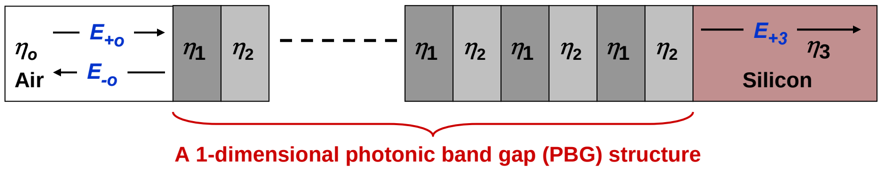
Consider HR coating on Silicon that is designed for keeping green light out of Silicon

One designs a HR coating for maximum reflectivity for green light at a frequency ω of 3.625×10^{15} rad/sec and a wavelength of 0.52 micro-meter in free-space

Question: How will the HR coating work for frequencies different from that of green light (e.g. for blue light, violet light, orange light, etc)



Photonic Band Gap (PBG) Structures



- **Photonic band gap (PBG) structures are structures in which the permittivity is periodic in space**
- **In PBG structures photons within certain bands of frequencies (called stop-bands) cannot propagate but are reflected off at the surface of the PBG structure**
- **The HR coating considered in the previous slide is an example of a 1-dimensional PBG structure**