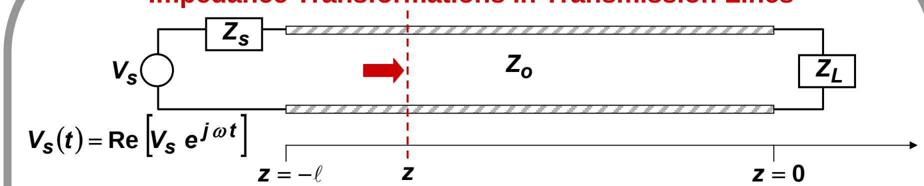
Lectur 22

RF and Microwave Circuit Design
Γ-Plane and Smith Chart Analysis

In this lecture you will learn:

- Γ-plane and Smith Charts
- Stub tuning
- Quarter-Wave transformers

Impedance Transformations in Transmission Lines



$$\Gamma_L = \frac{Z_L/Z_o - 1}{Z_L/Z_o + 1} \quad \text{and} \quad Z(z) = Z_o\,\frac{1+\Gamma_L\, e^{2jkz}}{1-\Gamma_L\, e^{2jkz}}$$

Define a position dependent reflection coefficient as:

$$\Gamma(z) = \Gamma_L\, e^{2jkz}$$

→ $\Gamma(z)$ is a complex number of magnitude never greater than unity

Define a normalized impedance as:

$$Z_n(z) = \frac{Z(z)}{Z_o} = R_n(z) + j\,X_n(z)$$

$R_n(z)$ → Normalized resistance

$X_n(z)$ → Normalized reactance

Then:

$$Z_n(z) = \frac{1+\Gamma(z)}{1-\Gamma(z)} \quad \text{or} \quad \Gamma(z) = \frac{Z_n(z)-1}{Z_n(z)+1}$$

Γ-Plane (Complex Plane)

$$\Gamma(z) = \Gamma_L \, e^{2jkz}$$

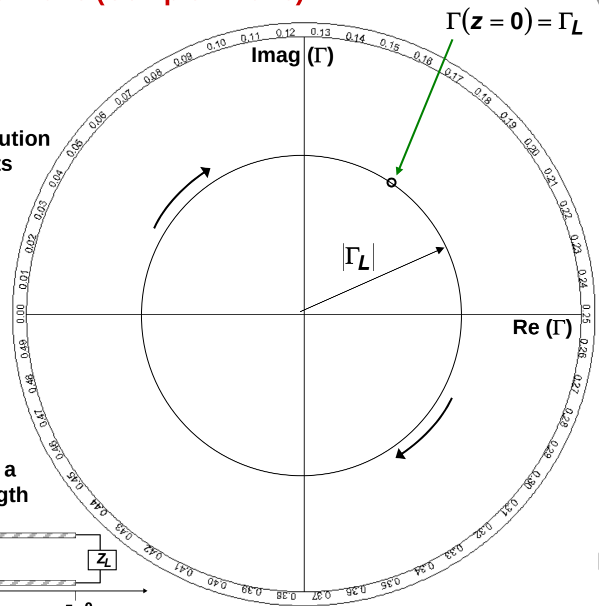
$\Gamma(z)$ **completes one full revolution in the complex plane when its phase goes through 2π**

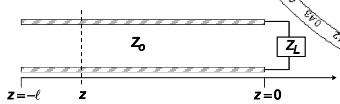
$$2kz = 2\pi$$

$$\Rightarrow \quad z = \frac{\lambda}{2}$$

$$Z_n(z) = \frac{1+\Gamma(z)}{1-\Gamma(z)}$$

Therefore, impedance is periodic with distance *z* with a period equal to half-wavelength



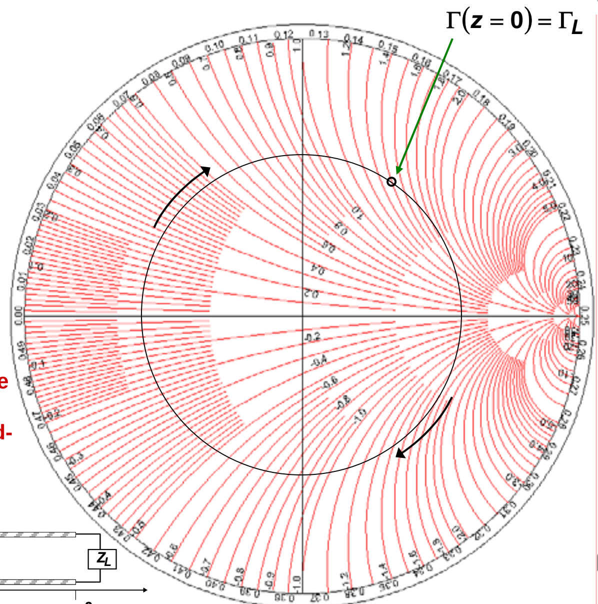


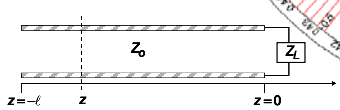
Γ-Plane with Normalized Reactance Curves

$$Z_n(z) = \frac{Z(z)}{Z_o} = R_n(z) + j\,X_n(z) = \frac{1+\Gamma(z)}{1-\Gamma(z)}$$

The red curves indicate the values of the normalized reactance $X_n(z)$ on the Γ-plane

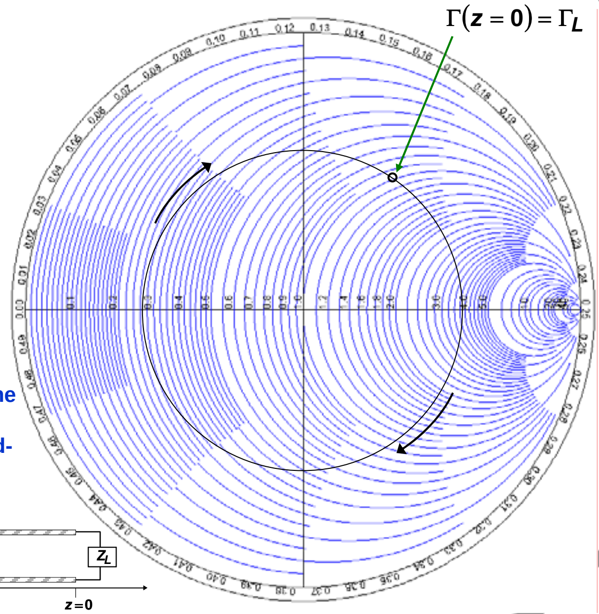
From the curves one can read-off the values of $X_n(z)$





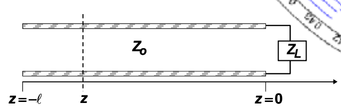
Γ-Plane with Normalized Resistance Curves

$$Z_n(z) = \frac{Z(z)}{Z_o}$$
$$= R_n(z) + j\,X_n(z)$$
$$= \frac{1+\Gamma(z)}{1-\Gamma(z)}$$



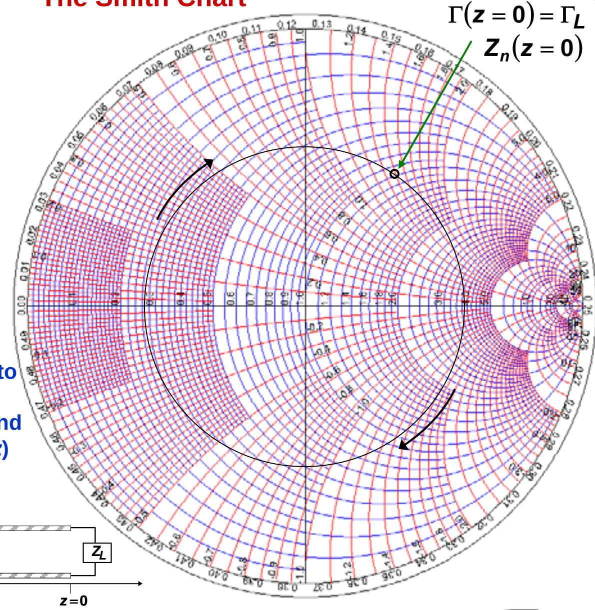
The blue curves indicate the values of the normalized resistance $R_n(z)$ on the Γ-plane

From the curves one can read-off the values of $R_n(z)$

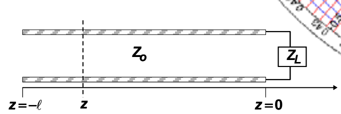


Γ-Plane with Normalized Resistance and Reactance Curves
The Smith Chart

$$Z_n(z) = \frac{Z(z)}{Z_o}$$
$$= R_n(z) + j\,X_n(z)$$
$$= \frac{1+\Gamma(z)}{1-\Gamma(z)}$$

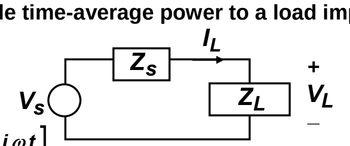


The Smith Chart enables one to read-off values of both the normalized resistance $R_n(z)$ and the normalized reactance $X_n(z)$ values on the Γ-plane



Load Matching for Maximum Power

In many microwave and RF circuits it is HIGHLY desirable to be able to transfer maximum possible time-average power to a load impedance:



$$V_s(t) = \text{Re}\left[V_s\, e^{j\omega t}\right]$$

$$\begin{cases} Z_s = R_s + j\,X_s \\ Z_L = R_L + j\,X_L \end{cases}$$

What is the time-average power delivered to the load?

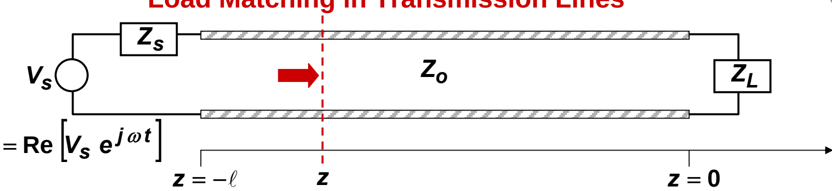
$$\langle P_L(t)\rangle = \langle V_L(t)\, I_L(t)\rangle = \frac{1}{2}\text{Re}\left[V_L\, I_L^*\right] = \frac{1}{2}\text{Re}\left[V_s \frac{Z_L}{Z_s + Z_L}\left(\frac{V_s}{Z_s + Z_L}\right)^*\right]$$

$$= \frac{1}{2}\frac{|V_s|^2}{|Z_s + Z_L|^2}\text{Re}\left[Z_L\right] = \frac{|V_s|^2}{2}\frac{R_L}{(R_s + R_L)^2 + (X_s + X_L)^2}$$

Maximum time-average power delivered to the load:

$\Rightarrow$ $\langle P_L(t)\rangle$ is maximized when: $\begin{cases} R_L = R_s \quad X_L = -X_s \\ \text{or} \\ Z_L = Z_s^* \end{cases}$ $\Rightarrow$ $\langle P_L(t)\rangle\big|_{max} = \dfrac{|V_s|^2}{8\,R_s}$

Load Matching in Transmission Lines

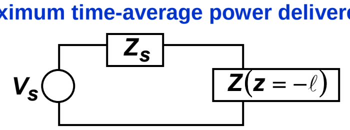


$$V_s(t) = \text{Re}\left[V_s\, e^{j\omega t}\right]$$

$$Z_n(z) = \frac{1 + \Gamma(z)}{1 - \Gamma(z)} \qquad Z(z) = Z_o\, Z_n(z) = Z_o\,\frac{1 + \Gamma(z)}{1 - \Gamma(z)}$$

How does one get the maximum time-average power delivered to the load?

The equivalent circuit is:

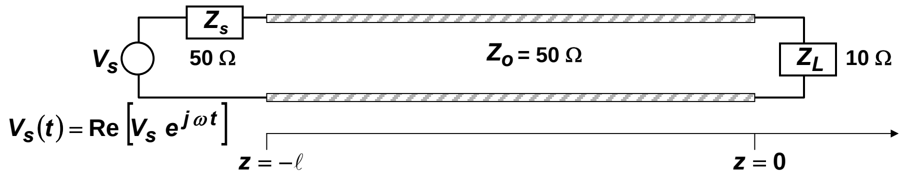


Power delivered to the load Z_L will be maximized if power delivered to the transformed impedance $Z(z=-\ell)$ is maximized

$\Rightarrow$ Must have the impedance $Z(z=-\ell)$ matched to the source impedance Z_s , i.e.

$$Z(z = -\ell) = Z_s^*$$

Example: Load Matching in Transmission Lines - I



Suppose: $Z_o = 50\ \Omega$ $\quad Z_L = 10\ \Omega$ $\quad Z_s = 50\ \Omega$

How does one get the maximum time-average power delivered to the load?

Need $Z(z=-\ell)$ matched to the source impedance $Z_s = 50\ \Omega$

STEP 1: First find Γ_L

$$\Gamma_L = \frac{Z_L/Z_o - 1}{Z_L/Z_o + 1} = -\frac{2}{3} = -0.667$$

STEP 2: Find "ℓ" such that the impedance $Z(z=-\ell)$ has a real part of 50 Ω

$$Z(z=-\ell) = 50 + j\,X(z=-\ell) \quad \Rightarrow \quad Z_n(z=-\ell) = 1.0 + j\,X_n(z=-\ell)$$

Example: Load Matching in Transmission Lines - II

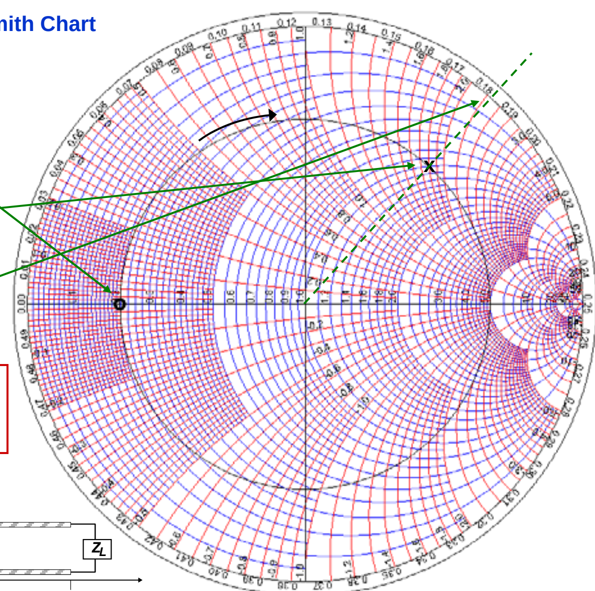
Find the desired "ℓ" using Smith Chart

$Z_n(z=0) = 0.2$

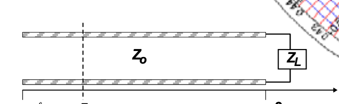
$\Gamma(z=0) = \Gamma_L = -0.667$

$Z_n(z=-\ell) \approx 1.0 + j1.8$

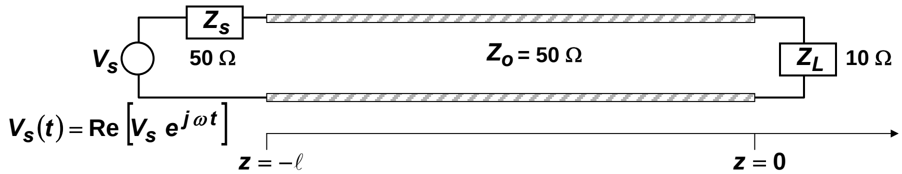
$\Rightarrow \quad \ell \approx 0.183\,\lambda$



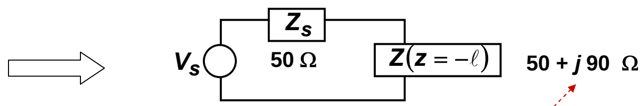
$$Z(z=-\ell) \approx (1.0 + j1.8)\,Z_o \approx 50 + j90$$



Example: Load Matching in Transmission Lines - III



So if $\ell = 0.183\,\lambda$ then the equivalent circuit is:



But we have a problem !

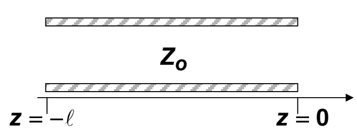
We have been able to match the real part of the impedance to the source but we ended up with an unwanted reactive term

How do we get rid of this unwanted reactance?

Answer: Use “stub tuners”

Stub Tuners in Microwave Circuits

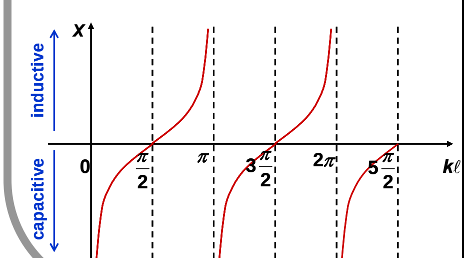
Stub tuners, as the name suggests, are short stubs of transmission lines that are used to cancel out unwanted reactances in microwave circuits

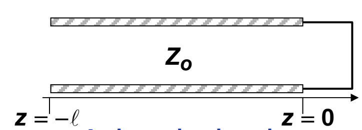


An open-circuit stub

$$Z(z = -\ell) = j\,X = -Z_o\; j\cot(k\,\ell)$$

It has a capacitive impedance for short lengths and is used to cancel unwanted inductive reactances

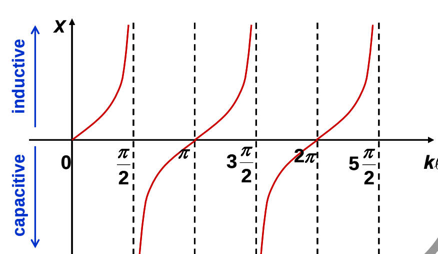




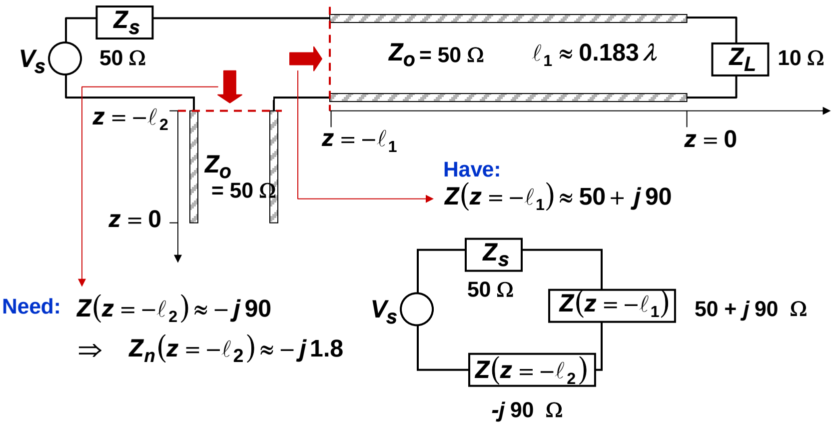
A short-circuit stub

$$Z(z = -\ell) = j\,X = Z_o\; j\tan(k\,\ell)$$

It has an inductive impedance for short lengths and is used to cancel unwanted capacitive reactances



Load Matching Using Series Stub Tuners



What length ℓ_2 of the stub must be chosen to get a reactance of $-j\,90$?

$$Z(z = -\ell_2) = -Z_o\ j\cot(k\,\ell_2) = -j\,90$$

$$\Rightarrow \quad k\ell_2 \approx 0.50$$

$$\Rightarrow \quad \ell_2 \approx .08\,\lambda$$

Load Matching Using Stub Tuners: Smith Chart

Open-Circuit stub tuner on a Smith Chart

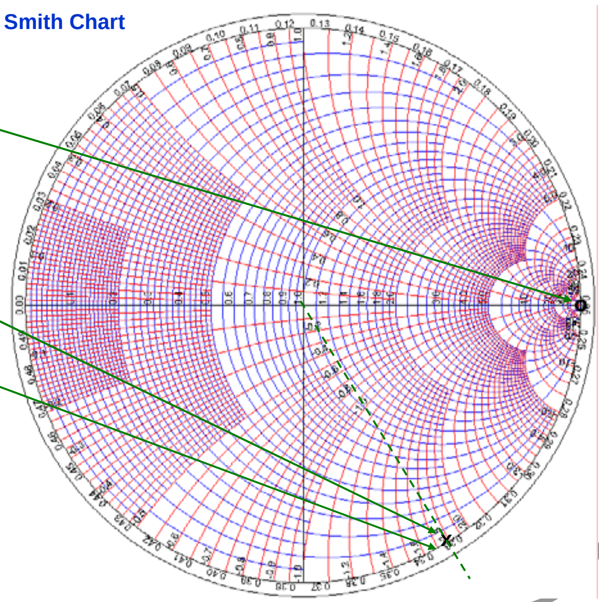
$$Z_n(z = 0) = \infty$$

$$\Gamma(z = 0) = \Gamma_L = +1$$

$$Z(z = -\ell_2) \approx -j\,90$$

$$\Rightarrow \quad Z_n(z = -\ell_2) \approx -j1.8$$

$$\Rightarrow \quad \ell_2 \approx 0.08\,\lambda$$



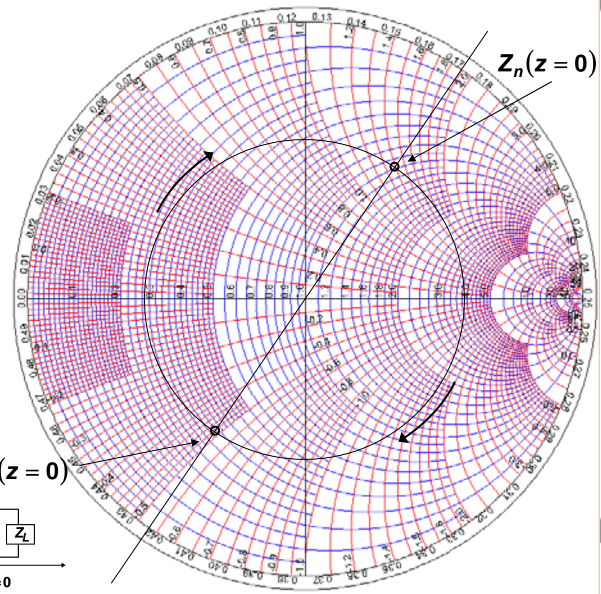
Admittances and Normalized Admittances on Smith Charts

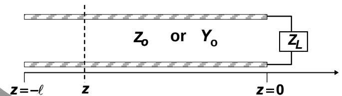
$$Z_n(z) = \frac{Z(z)}{Z_o} = R_n(z) + j\,X_n(z) = \frac{1+\Gamma(z)}{1-\Gamma(z)}$$

$$Y_o = \frac{1}{Z_o}$$

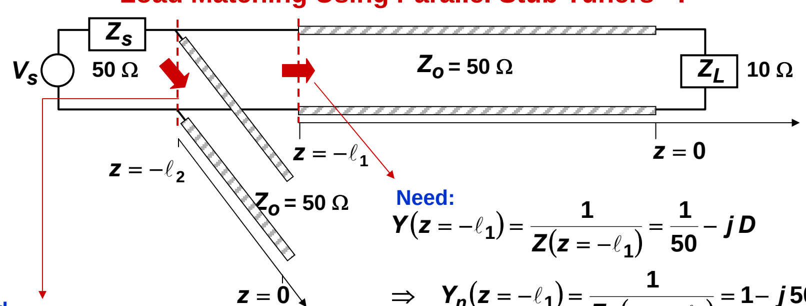
$$Y(z) = \frac{1}{Z(z)}$$

$$Y_n(z) = \frac{Y(z)}{Y_o} = \frac{1}{Z_n(z)} = \frac{1-\Gamma(z)}{1+\Gamma(z)}$$





Load Matching Using Parallel Stub Tuners - I



Need:

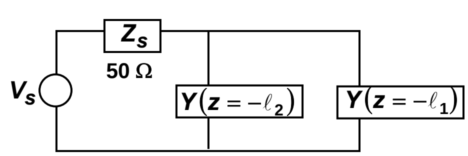
$$Y(z=-\ell_1) = \frac{1}{Z(z=-\ell_1)} = \frac{1}{50} - j\,D$$

$$\Rightarrow \quad Y_n(z=-\ell_1) = \frac{1}{Z_n(z=-\ell_1)} = 1 - j\,50\,D$$

Need:

$$Y(z=-\ell_2) = \frac{1}{Z(z=-\ell_2)} = +j\,D$$

$$\Rightarrow \quad Y_n(z=-\ell_2) = \frac{1}{Z_n(z=-\ell_2)} = +j\,50\,D$$

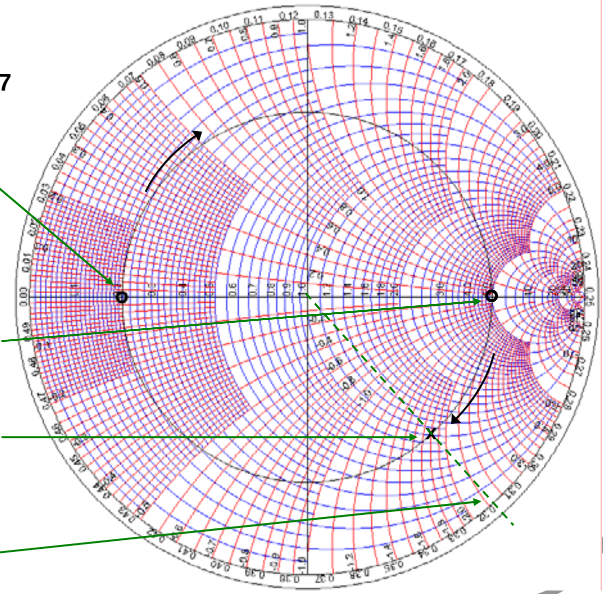


It is more useful here to work with admittances rather than impedances when one uses parallel stubs since admittances of circuit elements in parallel add

Load Matching Using Parallel Stub Tuners - II

$$Z_n(z=0)=\frac{10}{50}=0.2$$

$$\Gamma(z=0)=\Gamma_L=-0.667$$



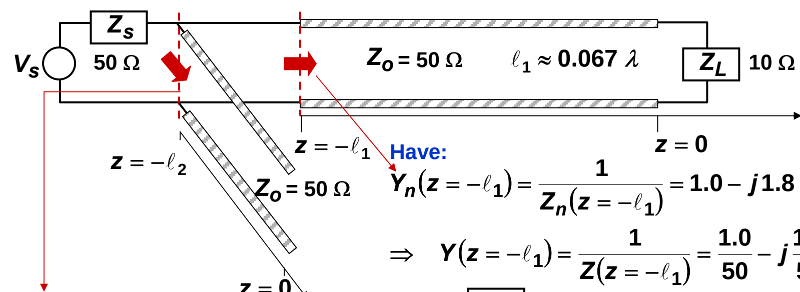
$$Y_n(z=0)=\frac{1}{Z_n(z=0)}=5.0$$

$$Y_n(z=-\ell_1)=\frac{1}{Z_n(z=-\ell_1)}=1.0-j\,1.8$$

$$\Rightarrow \quad \ell_1 \approx 0.067\,\lambda$$

Load Matching Using Parallel Stub Tuners - III

So what do we have so far:



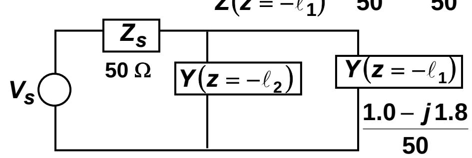
Have:

$$Y_n(z=-\ell_1)=\frac{1}{Z_n(z=-\ell_1)}=1.0-j\,1.8$$

$$\Rightarrow \quad Y(z=-\ell_1)=\frac{1}{Z(z=-\ell_1)}=\frac{1.0}{50}-j\frac{1.8}{50}$$

Need:

$$Y(z=-\ell_2)=\frac{1}{Z(z=-\ell_2)}=+\frac{j\,1.8}{50}$$



What length ℓ_2 of the stub must be chosen to get a reactance of +*j* 1.8/50?

$$Y(z=-\ell_2)=\frac{1}{Z(z=-\ell_2)}=j\frac{\tan(k\,\ell_2)}{Z_o}=j\frac{1.8}{50}$$

$$\Rightarrow \quad k\ell_2 \approx 1.064 \qquad \Rightarrow \quad \ell_2 \approx 0.17\,\lambda$$

Load Matching Using Parallel Stub Tuners - IV

Open-Circuit stub tuner on a Smith Chart

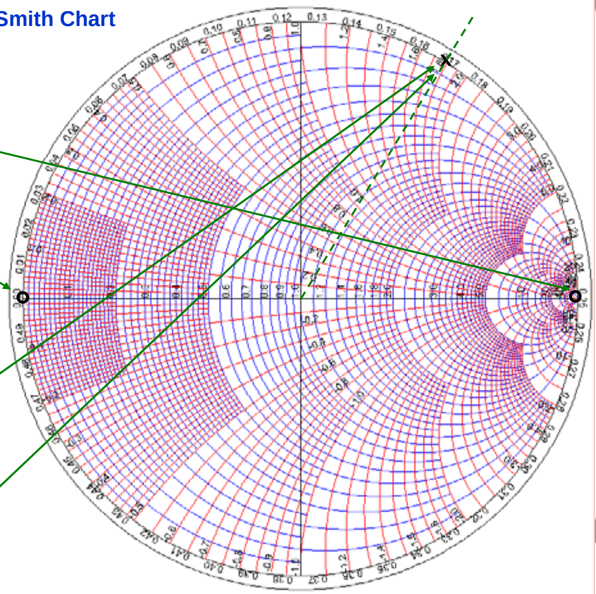
$$Z_n(z=0)=\infty$$

$$\Gamma(z=0)=\Gamma_L=+1$$

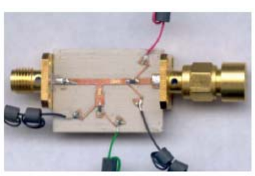
$$Y_n(z=0)=\frac{1}{Z_n(z=0)}=0$$

$$Y_n(z=-\ell_2)=\frac{1}{Z_n(z=-\ell_2)}=+j1.8$$

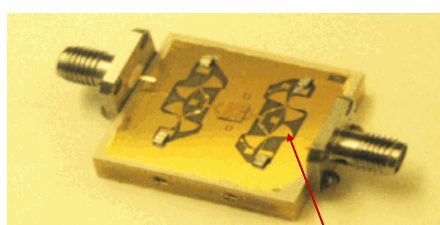
$$\Rightarrow \quad \ell_2 \approx 0.17\,\lambda$$



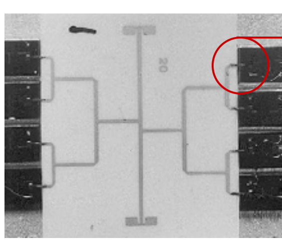
Stubs in Microwave Circuits: Some Pictures



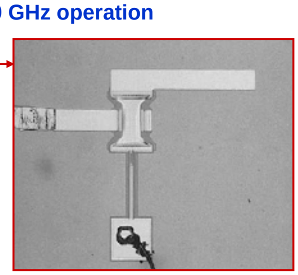
Photograph of an 10 GHz electronic single-stub tuner with varactor diode tuning elements



A GaN amplifier chip with radial stubs for 10 GHz operation

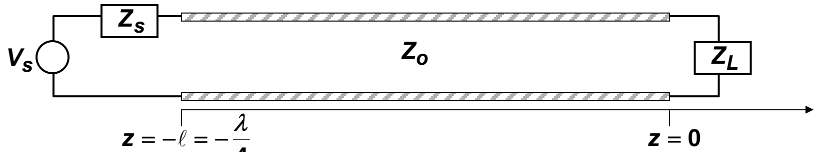


20 GHz micro-fabricated reconfigurable stub tuner



20 GHz micro-fabricated MEMS-switched stub tuner

Quarter-Wave Transformer



$$\Gamma_L = \frac{Z_L/Z_o - 1}{Z_L/Z_o + 1} \qquad Z_n(z) = \frac{Z(z)}{Z_o} = \frac{1+\Gamma_L e^{2jkz}}{1-\Gamma_L e^{2jkz}}$$

Suppose the length ℓ of the transmission line is quarter-wavelength:

$$\ell = \frac{\lambda}{4} \quad \Rightarrow \quad k\ell = \frac{\pi}{2}$$

$$\Rightarrow \quad Z_n(z=0) = \frac{Z_L}{Z_o} = \frac{1+\Gamma_L}{1-\Gamma_L}$$

$$\Rightarrow \quad Z_n\left(z = -\ell = -\frac{\lambda}{4}\right) = \frac{1+\Gamma_L e^{-2jk\ell}}{1-\Gamma_L e^{-2jk\ell}} = \frac{1-\Gamma_L}{1+\Gamma_L} = \frac{1}{Z_n(z=0)} = \frac{Z_o}{Z_L}$$

A quarter-wavelength long transmission line inverts the normalized impedance

Actual impedance at $z = -\lambda/4$: $Z\left(z = -\frac{\lambda}{4}\right) = Z_o Z_n\left(z = -\frac{\lambda}{4}\right) = \frac{Z_o^2}{Z_L}$

Quarter-Wave Transformer: On a Smith Chart

Suppose:

$Z_L = 50 + j50$

$Z_o = 50$

$$\Rightarrow \quad Z_n(z=0) = \frac{Z_L}{Z_o} = \frac{1+\Gamma_L}{1-\Gamma_L}$$
$$= 1.0 + j1.0$$

$$\Rightarrow \quad Z_n\left(z = -\frac{\lambda}{4}\right) = \frac{1}{Z_n(z=0)}$$
$$= 0.5 - j0.5$$

$$Z\left(z = -\frac{\lambda}{4}\right) = Z_o Z_n\left(z = -\frac{\lambda}{4}\right)$$
$$= 25 - j25$$