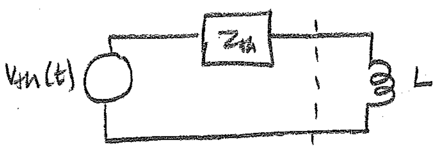
ECE 303 Prelim #3 Solutions.

Problem 1 a) See attached Solutions.

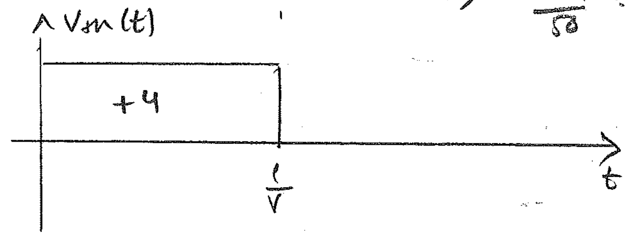
b)



$Z_{th} = Z_0 = 50\ \Omega$.

For $t<0$ $V_+ = -V_-$ & $I_+ + I_- = I = \frac{4}{Z_s} = \frac{4}{50}$

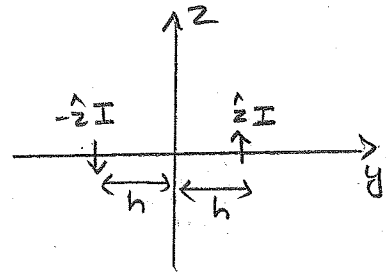
$\Rightarrow \frac{2V_+}{50} = \frac{4}{50} \Rightarrow V_+ = 2V \Rightarrow V_- = -2V$



For $t \geq 0$, the forward going V_+ of 2 Volts would reflect as +2 Volts (because of open) to give +4 Volts at $z=0$ for $0 \leq t \leq \frac{\ell}{v}$.

c) See attached Solutions d) See attached Solutions.

Problem 2. The dipole will have an image as shown



a) $\vec{E}_{ff}(\vec{r}) = \hat{\theta}\, j\, \eta_0 \frac{kId}{4\pi r} \sin\theta\, e^{-jkr} \left\{ e^{jkh\sin\theta\sin\phi} - e^{-jkh\sin\theta\sin\phi} \right\}$.

$= -\hat{\theta}\, \eta_0 \frac{kId}{2\pi r} e^{-jkr} \sin\theta \sin\left[kh\sin\theta\sin\phi\right]$ {for $y \geq 0$}

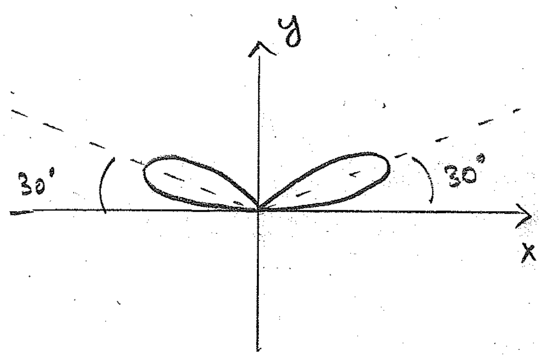
b) $P = \frac{\eta_0}{12\pi}|kId|^2 \longrightarrow$ It is the same as that of a SINGLE hertzian dipole.

c) $P(\theta,\phi) \propto \sin^2\theta \sin^2\left[kh\sin\theta\sin\phi\right]$

d) See the sketch. $P(\theta=\frac{\pi}{2},\phi) = \sin^2\left[\pi\sin\phi\right]$

max. when $\sin\phi = +\frac{1}{2} \Rightarrow \phi = 30$-degrees & 150-degrees

null when $\sin\phi = +1, 0$
ϕ = 90-degrees
= 0-degrees
= 180-degrees



Problem 3.

a) Starting from right → first layer is half-wavelength ⇒ does not transform impedance → third layer also does not transform impedance ⇒ can think of two quarter-wavelength slabs sitting back-to-back – but that is just one more half-wavelength slab which would not transform impedance $\Rightarrow \Gamma = 0$.

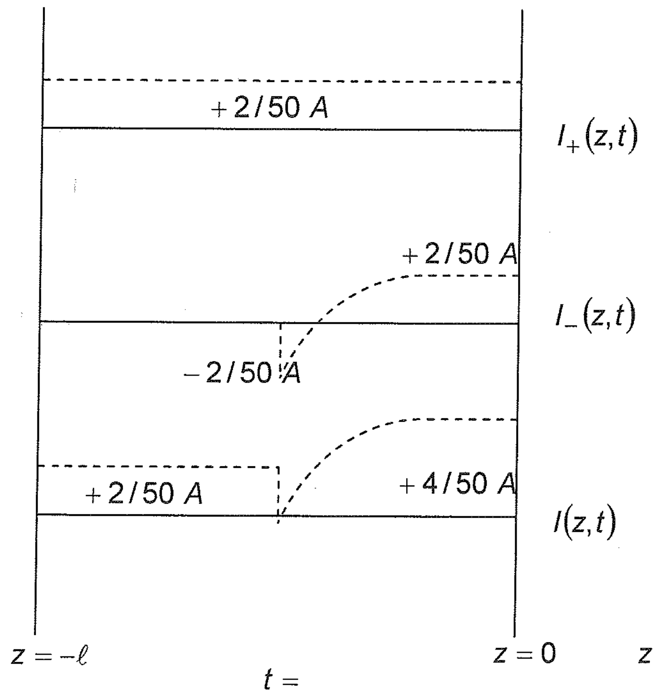
b) $\Gamma(z) = \Gamma_L e^{2jkz} = \frac{1}{5}e^{2jkz} \Rightarrow \Gamma(-\ell) = \frac{1}{5}e^{-j\frac{\pi}{2}} = -j/5. \Rightarrow \eta_n(-\ell) = \frac{1+\Gamma(-\ell)}{1-\Gamma(-\ell)} = \frac{5-j}{5+j}$

$\Rightarrow \eta(-\ell) = \frac{\eta_0}{3}\left(\frac{5-j}{5+j}\right) \Rightarrow \Gamma = \frac{\eta(-\ell)-\eta_0}{\eta(-\ell)+\eta_0} = -\frac{5+2j}{10+j}$

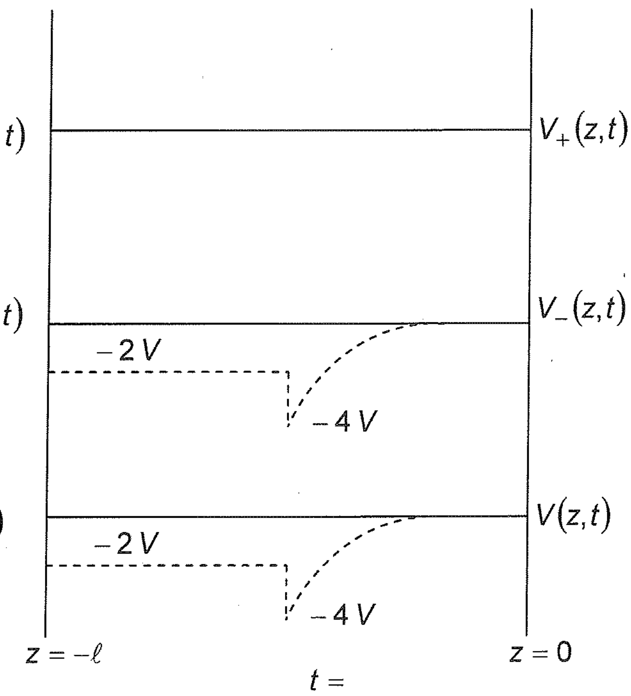
{ I have assumed that z=0 point is z=−ℓ point to use transmission line formula }

c) Energy Conservation $\Rightarrow \frac{(1-|\Gamma|^2)}{\eta_0} = \frac{|T|^2}{\eta_0/2} \Rightarrow |T|^2 = \frac{1}{2}(1-|\Gamma|^2)$

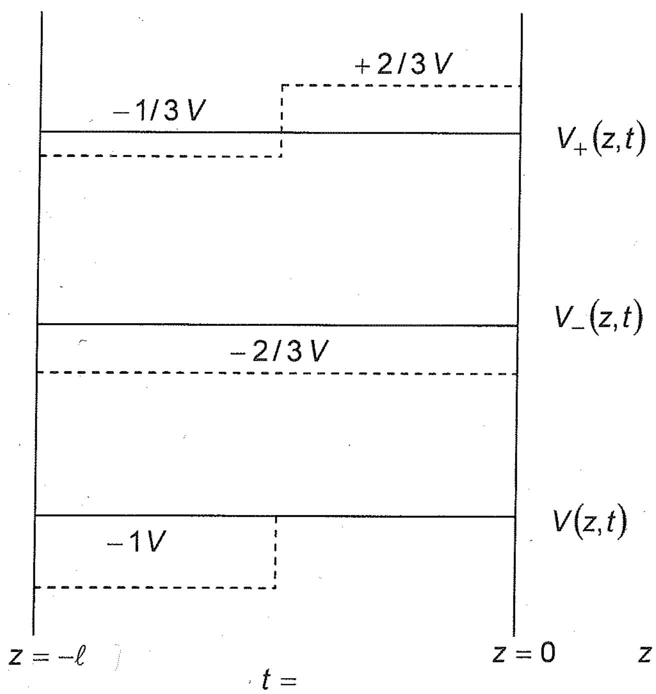
Problem and Part Number: 1 (a)



Problem and Part Number: 1 (c)



Problem and Part Number: 1 (d)



Problem and Part Number: