ECE 303 Exam #2 Solutions (Farhan Rana)

P1:

a) The wave is a superposition of a TE and TM wave. The TE part is $E_o(2\hat{y})\,e^{-j\vec{k}_i\cdot\vec{r}}$ and the TM part is $E_o(-j\hat{x}+j\sqrt{3}\hat{z})e^{-j\vec{k}_i\cdot\vec{r}}$ $= -jE_o(\hat{x}-\sqrt{3}\hat{z})e^{-j\vec{k}_i\cdot\vec{r}}$. Also, the magnitudes of TE + TM components are the same and the TM component is lagging behind in phase by 90° $\Rightarrow$ left-hand circularly polarized wave.

b) Since for the TM wave, the polarization direction is along the unit vector $\left[\frac{\hat{x}}{2}-\frac{\sqrt{3}}{2}\hat{z}\right]$ which must equal $\left[\cos\theta_i\,\hat{x}-\sin\theta_i\,\hat{z}\right]$ $\Rightarrow \sin\theta_i=\frac{\sqrt{3}}{2}$. One can use Snell's law to find θ_t but note that the TM part sees an index of $\sqrt{3}$ and the TE part sees and index of $\sqrt{\frac{2}{3}}$. Therefore for the TM part we have:

$$n_i\sin\theta_i = n_t\sin\theta_t \Rightarrow \sin\theta_t^{TM}=\frac{1}{\sqrt{3}}\frac{\sqrt{3}}{2}=\frac{1}{2} \Rightarrow \theta_t^{TM}=\frac{\pi}{6}$$

For the TE part we have $n_i\sin\theta_i=n_t\sin\theta_t \Rightarrow \sin\theta_t^{TE}=\sqrt{\frac{3}{2}}\,\frac{\sqrt{3}}{2}=\sqrt{\frac{9}{8}}>1$
$\Rightarrow$ TE part is totally internally reflected. Only the TM part is transmitted.

c) $$\vec{E}_t(\vec{r}) = T_{TE}\left[E_o\,2\hat{y}\right]e^{-j\vec{k}_t^{TE}\cdot\vec{r}} + \left[E_o\left(-j\hat{x}\,T_{TM}^{x}+j\sqrt{3}\,\hat{z}\,T_{TM}^{z}\right)\right]e^{-j\vec{k}_t^{TM}\cdot\vec{r}}$$

$$\vec{k}_t^{TM}=\frac{\omega}{c}\sqrt{3}\left[\sin\theta_t^{TM}\,\hat{x}+\cos\theta_t^{TM}\,\hat{z}\right] \quad\text{and}\quad \vec{k}_t^{TE}=k_{tx}^{TE}\,\hat{x}+k_{tz}^{TE}\,\hat{z}$$

$$k_{tx}^{TE}=k_{ix}^{TE}=k_{ix}=\frac{\omega}{c}\sin\theta_i=\frac{\omega}{c}\frac{\sqrt{3}}{2} \Rightarrow k_{tz}^{TE}=\sqrt{\frac{\omega^2}{c^2}\frac{2}{3}-\left(k_{tx}^{TE}\right)^2}=-j\frac{\omega}{c}\frac{1}{2\sqrt{3}}$$

$$T_{TE}=\frac{2\,k_{iz}^{TE}/k_{tz}^{TE}}{k_{iz}^{TE}/k_{tz}^{TE}+1} \quad\text{and}\quad T_{TM}^{x}=\frac{2}{\sqrt{3}\dfrac{\cos\theta_i}{\cos\theta_t^{TM}}+1} \qquad T_{TM}^{z}=\frac{\dfrac{2}{\sqrt{3}}\dfrac{\cos\theta_i}{\cos\theta_t^{TM}}}{\sqrt{3}\dfrac{\cos\theta_i}{\cos\theta_t^{TM}}+1}$$

P2:

$$V(z)=V_+\left[e^{-jkz}+\Gamma_L e^{jkz}\right] \Rightarrow |V(z)|=|V_+|\sqrt{1+|\Gamma_L|^2+2|\Gamma_L|\cos(2kz+\phi)}$$

$\Gamma_L=|\Gamma_L|e^{j\phi}$

a) The variations in the plot have the periodicity of half the wavelength $\Rightarrow \frac{\lambda}{\sqrt{3}}=8\text{ mm} \Rightarrow \lambda=\sqrt{3}\,8\text{ mm} \Rightarrow f=\frac{c}{\lambda}=21.65\text{ GHz}$.

b) $SWR = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} = \frac{6}{2} = 3 \Rightarrow |\Gamma_L| = \frac{1}{2}$

c) The min value of $|V(z)|$ occurs for the 1st time when the term $(2kz+\phi)$ in $|V(z)| = |V_+|\sqrt{1+|\Gamma_L|^2 + 2|\Gamma_L|\cos(2kz+\phi)}$ becomes $-\pi$.

$$2kz+\phi = -\pi$$

$$\Rightarrow -2\left(\frac{2\pi}{\lambda}\right)3 + \phi = -\pi$$

$$\Rightarrow -\frac{3\pi}{2} + \phi = -\pi \quad \Rightarrow \phi = \frac{\pi}{2} \quad \Rightarrow \Gamma_L = \frac{j}{2}.$$

d) $|I_+| = \frac{|V_+|}{Z_0}$ $\quad |V_+| = \frac{|V(z)|_{max}}{1+|\Gamma_L|} = \frac{6}{1+\frac{1}{2}} = \frac{12}{3} = 4$ Volts

$\Rightarrow |I_+| = \frac{4}{50}$ Amps.

e) $Z_L = Z_0 \frac{1+\Gamma_L}{1-\Gamma_L} = 50\frac{(2+j)}{(2-j)}$

P3

a) ~~Need $Y(z=-l_1)$~~ to have real part of $\frac{1}{100} = .01$

$\Rightarrow Y_n(z=-l_1)$ has real part of 0.5. $\quad Y_n(z=0) = \frac{50}{10} = 5$

From Smith chart $l_1 = 0.105\lambda$ $\quad Y_n(z=-l_1) = 0.5 - 1.15j$

$\Rightarrow Y(z=-l_1) = \frac{0.5-1.15j}{50} = .01 - .023j$

b) Need $Y_n(z=-l_2) = (+.023j)\,100 = +2.3j$.

From Smith chart; $l_2 = 0.185\lambda$

c). $\frac{|V_+|^2}{Z_0} = \frac{|V_{+1}|^2}{Z_{01}}\left(1-|\Gamma_L|^2\right)$ $\quad \Gamma_L = \frac{0.2-1}{0.2+1} = -\frac{2}{3}$ $\quad \Big\{$ power coming in must equal powe delivered to load

$\Rightarrow |V_{+1}| = \sqrt{\frac{50 \times 4}{100\left(1-\frac{4}{9}\right)}} = 1.897$ Volts.

d) when frequency is doubled, wavelength is reduced by half.

so $\lambda_{new} = \frac{\lambda}{2}$. or $\lambda = 2\lambda_{new}$.

We had $l_1 = 0.105\lambda = 0.21\ \lambda_{new} \Rightarrow Y_n(z=-l_1) = 0.22-0.25j$

$\Rightarrow\ Y(z=-l_1) = (0.22-0.25j)\,Y_0 = \frac{0.22-0.25j}{50}$