School of Electrical and Computer Engineering, Cornell University

ECE 303: Electromagnetic Fields and Waves

Fall 2007

Exam 2 **October 23, 2007**

INSTRUCTIONS:

- **Only work done on the blue exam booklets will be graded – do not attach your own sheets to the exam booklets under any circumstances**
- **Every problem must be done in a separate blue booklet – so you must have 3 separate blue booklets before starting the exam**
- **To get partial credit you must show all the relevant work**
- **Correct answers with wrong reasoning will not get points**
- **All questions do not carry equal points**
- **All questions do not have the same level of difficulty**

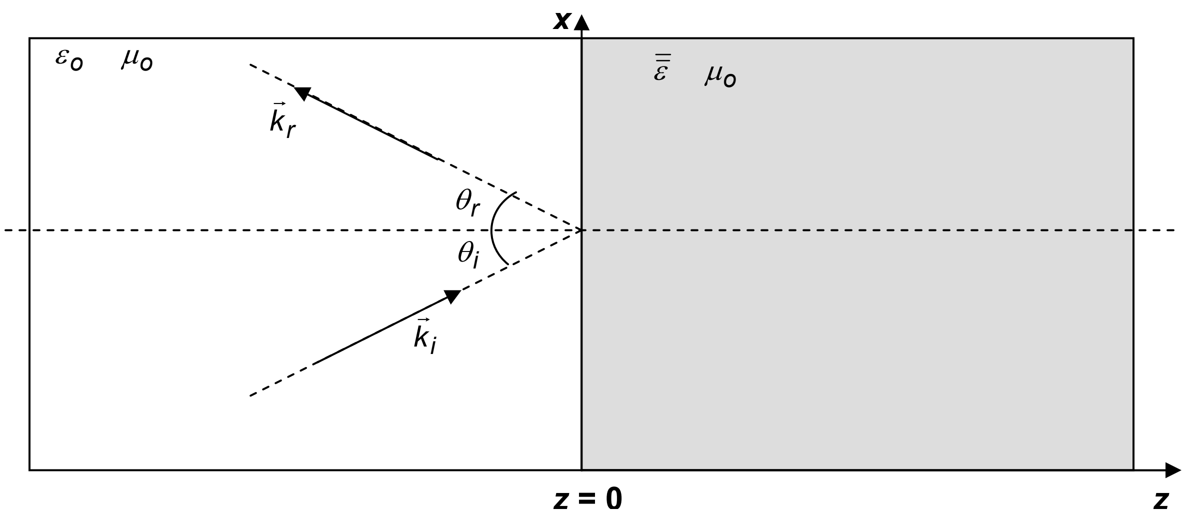
DO NOT WRITE IN THIS SPACE

Problem 1 (30 points)

Consider a plane wave given by:

$$\vec{E}_i(\vec{r}) = E_o\left(-j\,\hat{x} + 2\,\hat{y} + j\sqrt{3}\,\hat{z}\right)e^{-j\,\vec{k}_i\,.\,\vec{r}}$$

incident from free space upon a medium whose permittivity is described by the tensor $\bar{\bar{\varepsilon}}$. The frequency of the incident wave is ω. The plane of incidence is the x-z plane. The interface is at z=0 as shown below.



a) What is the polarization of the incident wave (linear, right or left circular, right or left elliptical, etc)? Explain your answer. No points will be awarded for a correct answer but a wrong or misleading or inadequate or incomplete explanation.

Now suppose that the dielectric permittivity tensor of the medium on the right is:

$$\bar{\bar{\varepsilon}} = \varepsilon_o\begin{bmatrix} 3 & 0 & 0 \\ 0 & 2/3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

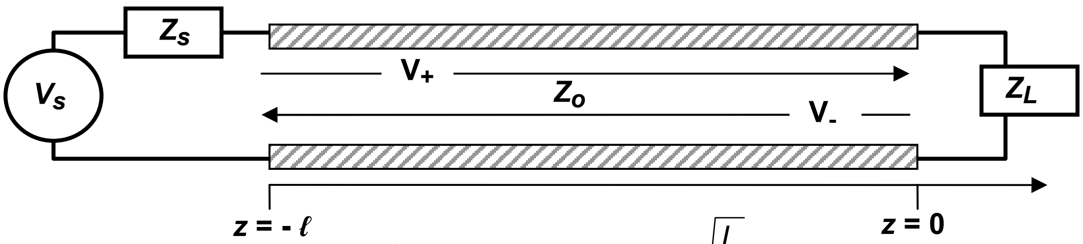
b) In which direction does the transmitted wave go? Specify using the angle of transmission θ_t and give numerical value.

c) Write the complete expression for the transmitted E-field $\vec{E}_t(\vec{r})$ in terms of the given parameters. There should be no unknown or unspecified variables in your answer. The values of all the variables must be specified in terms of the given parameters.

DO NOT WRITE IN THIS SPACE

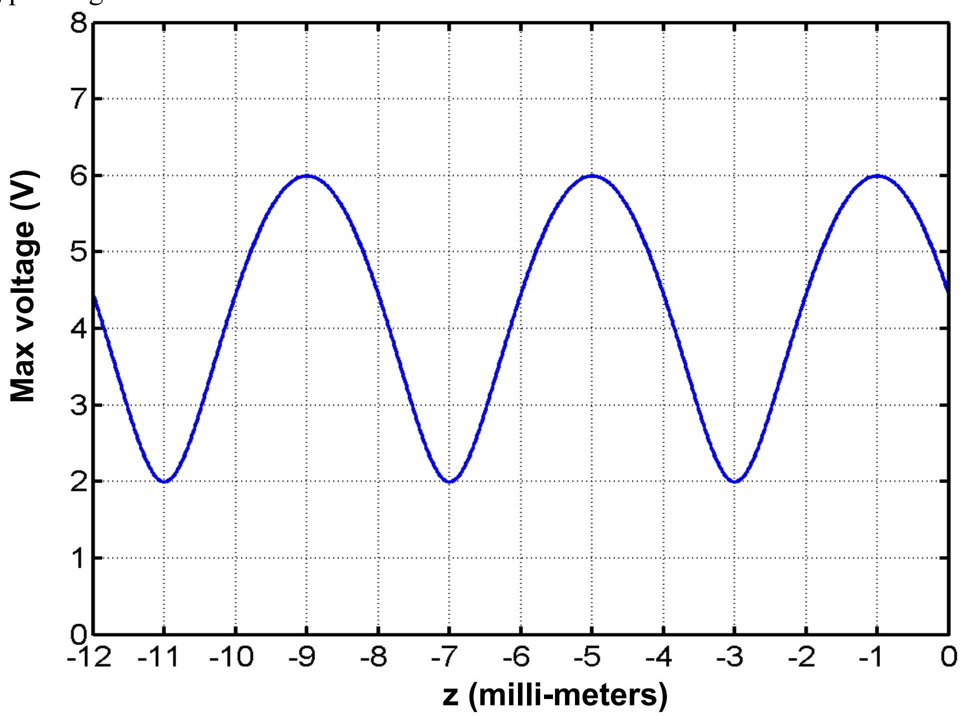
Problem 2 (30 points)

Consider a load connected to a sinusoidal voltage source via a dielectric filled transmission line, as shown below.



$$LC = \varepsilon\,\mu_o = 3\varepsilon_o\,\mu_o \qquad \sqrt{\frac{L}{C}} = 50\,\Omega$$

The maximum amplitude (single-sided - not peak to peak) of voltage oscillations $V(z,t)$ on the transmission line are measured at each location on the transmission line during operation and the following plot is generated.



a) What is the frequency of the voltage source in Hz? Give a numerical value.

b) What is **magnitude** $|\Gamma_L|$ of the load reflection coefficient? Give a numerical value.

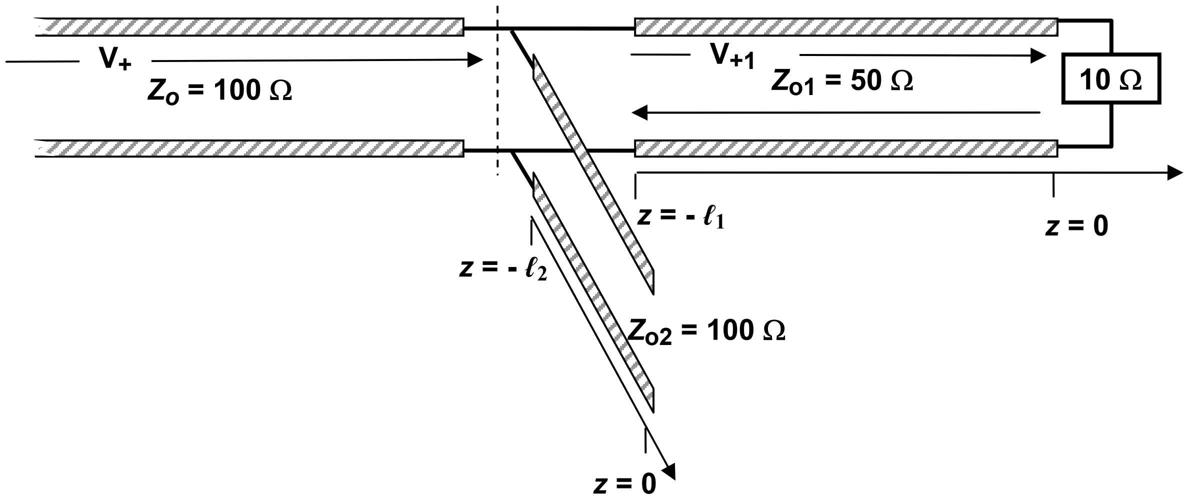
c) What is the **phase** of the load reflection coefficient Γ_L? Give a numerical value.

d) What is the magnitude $|I_+|$ of the forward current wave on the transmission line? Give a numerical value.

e) What is the value of the load impedance Z_L?

Problem 3 (40 points)

Consider the following transmission line circuit shown in the figure below. On the left is a transmission line carrying an input signal specified by the amplitude V_+ of the forward going voltage wave on that transmission line. Assume $|V_+| = 2.0$ Volts. On the right is a load impedance of 10 Ω. The goal is to transfer all the input power to the 10 Ω load impedance. You have at your disposal two design parameters - you can choose the lengths ℓ_1 and ℓ_2 of both the transmission lines. Assuming that the wavelength of the waves at the frequency of operation is λ in all the transmission lines, you need to specify the lengths ℓ_1 and ℓ_2 in terms of the wavelength λ. Use the attached smith charts that already have some circles drawn on it. You may find one or more of these circles useful. **Note that the impedances of all the transmission lines are not the same.**

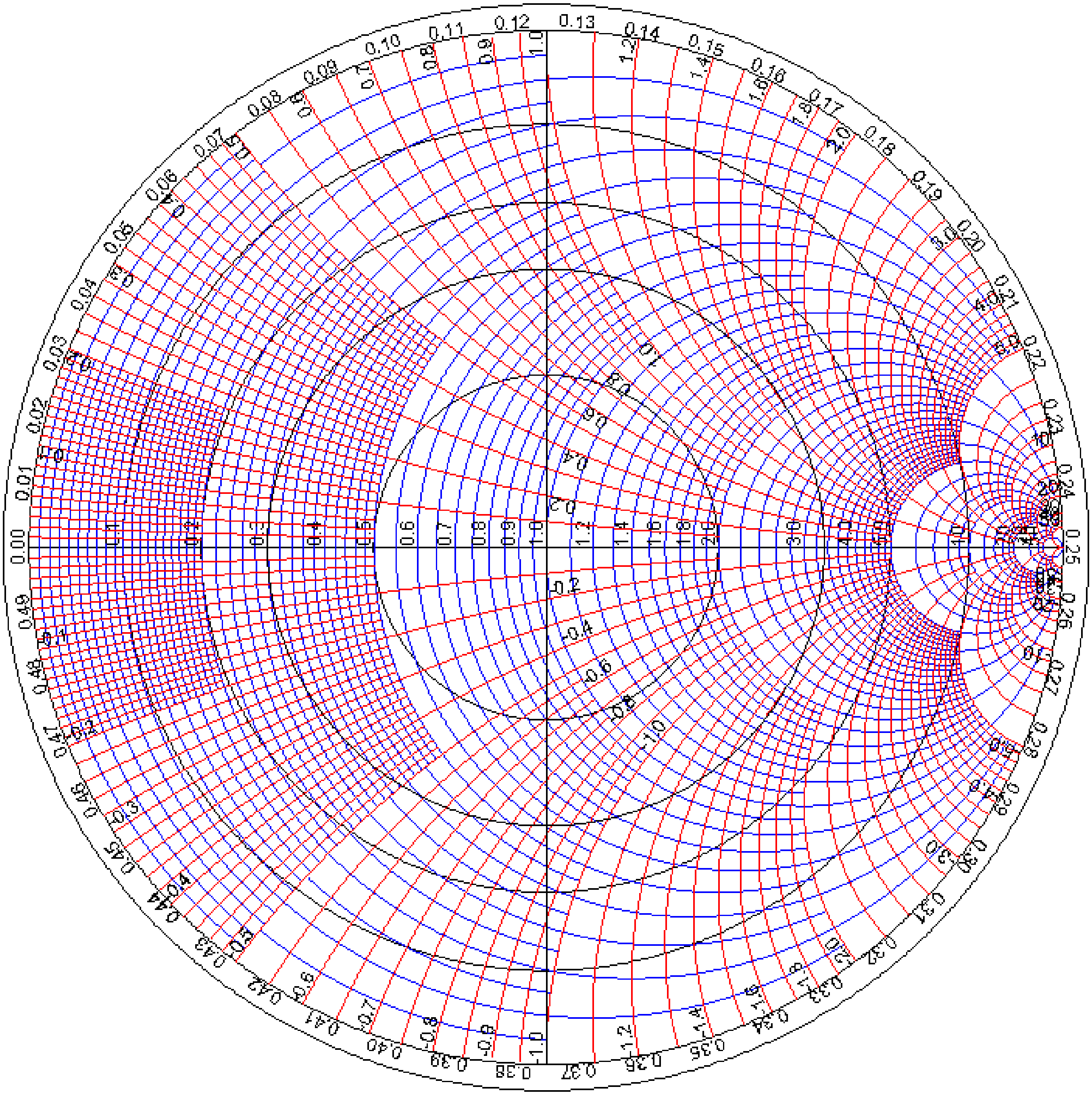


a) Find the smallest length ℓ_1 in terms of the wavelength λ such that you get the right impedance/admittance at $z = -\ell_1$ that will enable you to accomplish the task at hand. Show your work on the attached smith chart and point out all the important points (start point, finish point, direction of motion). Also explain how should one select the appropriate value of the length ℓ_1.

b) Find the smallest length ℓ_2 in terms of the wavelength λ such that you get the right impedance/admittance at $z = -\ell_2$ that will enable you to accomplish the task at hand. Show your work on the attached smith chart and point out all the important points (start point, finish point, direction of motion).

c) Suppose your structure has been designed correctly and is working as desired. What is the magnitude $|V_{+1}|$ of the forward going voltage wave in the transmission line on the right?

d) Suppose you have designed your matching circuit above at a frequency of 9 GHz, and it works the right way. Using your answers from part (a) above, figure out using the Smith chart what will be the value of the impedance (or admittance) at $z = -\ell_1$ if your designed structure remained the same but the frequency was cranked up to be 18 GHz. Show your work on the attached smith chart and point out all the important points.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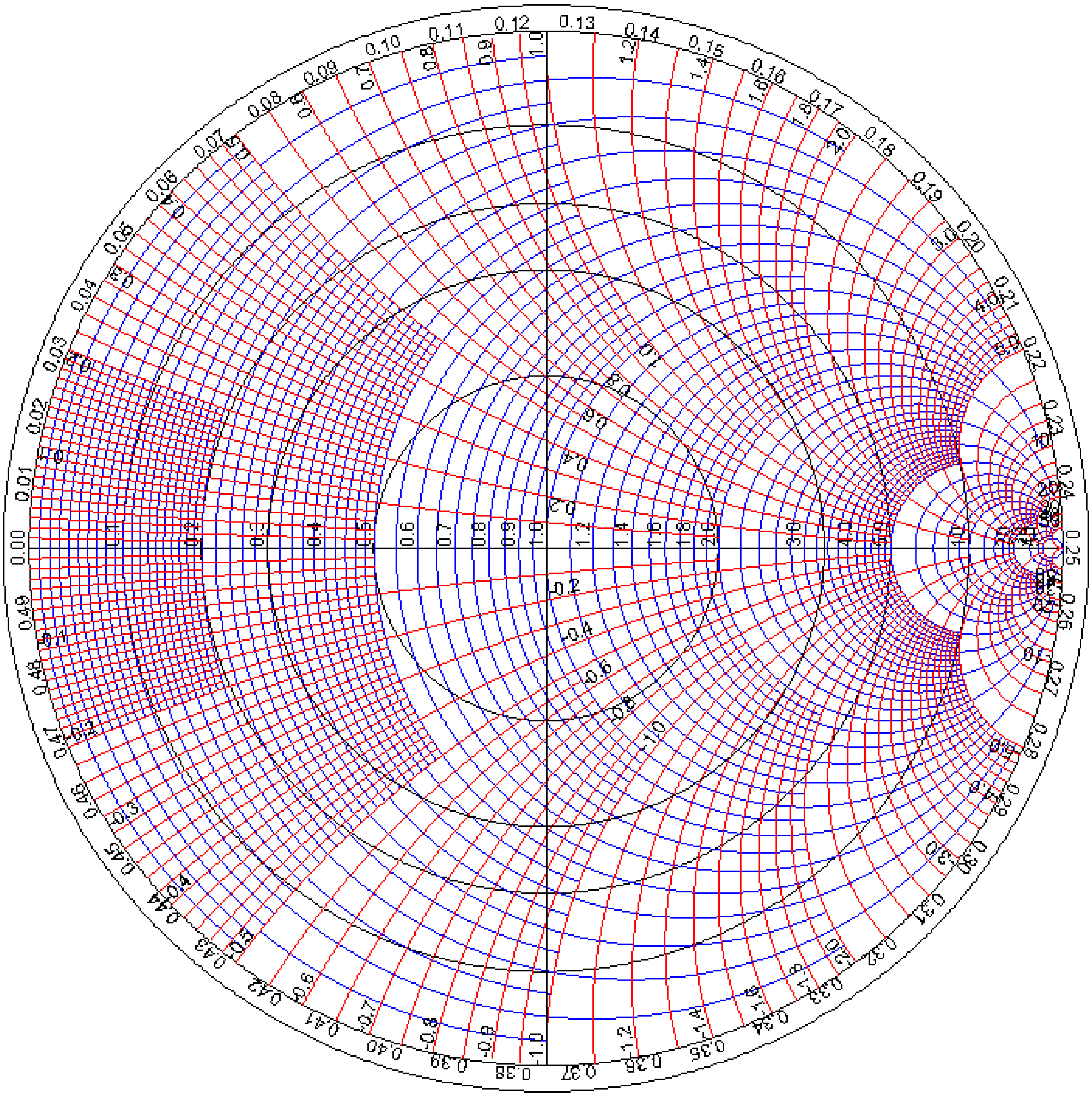

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The Complete Smith Chart