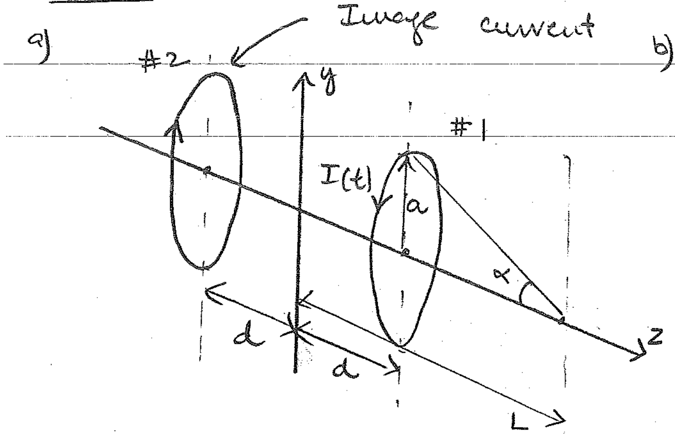
ECE 303 Exam #1 Solutions (Farhan Rana)

Problem 1

a)



b) For current loop #1:

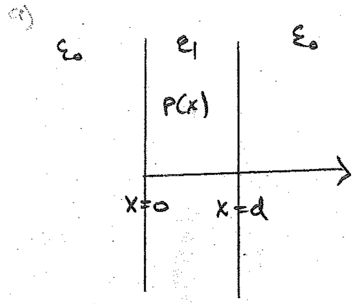
$$H_z = \frac{I(t)}{4\pi}\,\frac{2\pi a}{a^2+(L-d)^2}\cdot \sin\alpha = \frac{I(t)}{2}\,\frac{a^2}{\left[a^2+(L-d)^2\right]^{3/2}}$$

For current loop #1 and #2:

$$H_z = \frac{I(t)a^2}{2}\left\{\frac{1}{\left[a^2+(L-d)^2\right]^{3/2}} - \frac{1}{\left[a^2+(L+d)^2\right]^{3/2}}\right\}$$

c) By symmetry the x-component of H field will be zero.

Problem 2



a) $E_x(x=d^+) = E_0$ Use boundary condition: $\varepsilon_0 E_x(x=d^+) = \varepsilon_1 E_x(x=d^-)$

$$\Rightarrow E_x(x=d^-) = \frac{\varepsilon_0}{\varepsilon_1}E_0$$

Gauss law $\Rightarrow$ $\dfrac{\partial\, \varepsilon_1 E_x(x)}{\partial x} = \rho(x)$ for $0<x<d$

Integrate from x to d^- both sides

$$\varepsilon_1 E_x(x=d^-) - \varepsilon_1 E_x(x) = \int_x^{d^-} \rho(x')\,dx' = \frac{\rho_0 d}{3}\left[1-\frac{x^3}{d^3}\right]$$

$$\Rightarrow E_x(x) = \frac{\varepsilon_0 E_0}{\varepsilon_1} - \frac{\rho_0 d}{3\varepsilon_1}\left[1-\frac{x^3}{d^3}\right] \quad \text{for } 0<x<d.$$

b) Use boundary condition $\varepsilon_0 E_x(x=0^-) = \varepsilon_1 E_x(x=0^+) = \varepsilon_0 E_0 - \dfrac{\rho_0 d}{3}$

$$\Rightarrow E_x(x=0^-) = E_0 - \frac{\rho_0 d}{3\varepsilon_0} \quad \text{since } \frac{\partial\, \varepsilon_0 E_x(x)}{\partial x} = 0 \text{ for } x<0$$

$$\Rightarrow E_x(x) = E_0 - \frac{\rho_0 d}{3\varepsilon_0} \quad \text{for } x<0.$$

c) $\sigma_p(x=d) = \varepsilon_0 E_0\left(1-\dfrac{\varepsilon_0}{\varepsilon_1}\right)$ $\quad \sigma_p(x=0) = -\left(\varepsilon_0 E_0 - \dfrac{\rho_0 d}{3}\right)\times\left(1-\dfrac{\varepsilon_0}{\varepsilon_1}\right)$

Problem 3

a) $\phi_{out}(\vec r) = 0$ (perfect infinite metal) $\quad \phi_{in}(\vec r) = \dfrac{P_0}{4\pi\varepsilon_0}\dfrac{\cos\theta}{r^2} + \underbrace{Br\cos\theta}$

b) Boundary condition $\phi_{in}(r=a) = \phi_{out}(r=a) \Rightarrow \dfrac{P_0}{4\pi\varepsilon_0 a^2} + Ba = 0.$

$$\Rightarrow B = -\frac{P_0}{4\pi\varepsilon_0 a^3} \qquad \Rightarrow \phi_{in}(\vec r) = \frac{P_0\cos\theta}{4\pi\varepsilon_0 r^2} - \frac{P_0}{4\pi\varepsilon_0 a^2}\left(\frac{r}{a}\right)\cos\theta.$$

due to induced surface charge density.

c) $$\sigma = -\varepsilon_0 E_r(r=a) = \varepsilon_0 \left.\frac{\partial \phi_{in}}{\partial r}\right|_{r=a} = -\frac{3P_0\cos\theta}{4\pi a^3}$$

d)